

FRACTURE MECHANICS APPLIED TO ROCK BLASTING

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ABSTRACT

Various expansion loaded radial crack systems are discussed in Fracture Mechanics terms. The influence of the blast hole and of the gas penetration into the cracks on the crack tip stresses and on the crack growth are treated.

Radial crack systems are found to have an inherent tendency towards non-uniform growth. Hereby the growth of all but a few of the original cracks from a blast hole is suppressed. This mechanism is largely insensitive to gas penetration into the crack system and it involves crack tip closure of short cracks. However, the crack lengths one may obtain in rock blasting are strongly influenced by the gas penetration.

RESUME

On discute un nombre de systèmes à fissures radiales, chargées des forces d'expansion, aux termes de la mécanique de rupture. On a expliqué comment agissent le trou de forage et la pénétration du gaz d'explosion dans les fissures sur les contraintes à l'extrémité de la fissure ainsi que l'extension.

Des systèmes à fissures radiales ne peuvent pas se développer uniformément. Ceci fait que l'extension de la plupart des fissures qui se sont produit auprès du trou de forage sera retenu. Ce mécanisme ne dépend pas en substance de la pénétration du gaz dans les fissures et comprend la fermeture des extrémités des fissures courtes. La pénétration du gaz exerce cependant une influence puissante sur les longueurs des fissures que l'on peut obtenir au creusement des roches.

ZUSAMMENFASSUNG

Eine Reihe durch Expansionskräfte belasteter radiale Rissysteme werden mit von der Bruchmechanik her bekannter Begriffsbildung behandelt. Insbesondere wird diskutiert wie das Bohrloch selbst und das Eindringen des Gases in die Risse auf die Spannungen an den Risspitzen sowie die Rissausbreitung einwirken.

Es wird gezeigt, dass radiale Rissysteme sich nur in Ausnahmefällen gleichförmig ausbreiten können. Hierbei werden sich nur wenige der am Bohrlochrande ursprünglich entstandenen Rissen weiter ausbreiten. Dieser Mechanismus ist im wesentlichen unabhängig vom Eindringen des Gases in die Risse und beinhaltet das Schliessen der Spitzen der kürzeren Risse. Die Risslängen, die beim Gesteinsprengen zu erreichen sind, werden dagegen vom Eindringen des Gases stark beeinflusst.

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INTRODUCTION

The difficulties associated with analyzing the rapid fracture process in rock blasting and the different situations to which the analyses were to apply have lead scientists to different angles of approach. Duvall and Atchison (1957) for example have emphasized dynamic strain wave effects in the formation of explosive craters. Others such as Langefors and Kihlström (1963) have chosen a quasistatic approach when stressing the gas pressure action in bench blasting. In more recent work by Persson and co-workers (1970) and by Kutter and Fairhurst (1971) these aspects are combined in a model of the fragmentation process in blasting where both play an essential role.

A quasi-static analysis of the fracture process in rock blasting is obviously not sufficient to explain the detailed course of events. But often it does capture essential information. The study of how a free surface influences the crack pattern around a bore hole is one example. Compare the quasi-static analysis of Porter and Fairhurst (1970) with the work of Field and Ladegaard-Pedersen (1971) on the importance of the reflected shock wave. In practise, however, one is often restricted to considering dynamic effects in a qualitative or quasi-static way.

The fragmentation of the rock in blasting is mainly a result of crack growth. In Fracture Mechanics one has a tool which retains the properties of the crack tips. The applicability to rock was realized by Bieniawski (1967) and recently validated by Hardy and co-workers (1973). Ouchterlony (1971) has for example used this approach to slab blasting. The Physics involved in it is essentially the same as was outlined by Griffith (1921), whose formulation was preferred by Kutter (1971) and Da Gama and Nelson (1971). The subject of crack growth is discussed by Johansson and Persson (1974) in the General Report of this theme.

The aim of this paper is to present a discussion on the growth of radial cracks from a cylindrical blast hole, resulting from an application of Fracture Mechanics. Linear elastic material behaviour is assumed.

FRACTURE MECHANICS AND CRACKS

For an intact material one may use a material property, such as a critical stress level, to decide when fracture occurs. When the material contains macroscopic cracks one also needs a relevant field quantity with a corresponding critical value, which is a material property, to be able to predict crack growth.

For a brittle material the governing field quantity is the rate at which the strain energy U available at the crack tip can be released. The crack will grow if it exceeds the rate at which energy is consumed in the formation of new fracture surfaces. In Linear Fracture Mechanics notation we may write the criterion for crack growth

$$-\frac{dU}{da} > G_c. \quad (1)$$

Here a denotes the crack length, $-dU/da$ denotes the strain energy release rate, and G_c denotes the fracture toughness of the material. G_c is an extension of Griffith's surface energy concept to include for example plasticity effects. It was introduced by Irwin (1957). The reverse criterion is used to determine crack arrest from a system of propagating cracks. However, the arrest value of G_c is usually somewhat lower than the initiation value.

Let us make assumptions about the crack system. First, the blast hole and the cracks are considered to be infinitely long. Secondly the crack system is assumed to be static and subject only to opening mode or mode I loads. This yields a material in plane deformation. See fig.1..

Hence the energy release rate may be expressed

$$-\frac{dU}{da} = \frac{1-\nu^2}{E} \cdot K_I^2(a) \quad (2)$$

The material is described by its Young's modulus E and its Poisson's ratio ν . The stress intensity factor, denoted by K_I , is thus a measure of the energy flow into the crack tip at a potential extension of the crack in its own plane. Its magnitude depends on the geometry of the crack system and on the load. It is closely related to the stresses and the displacements near the crack tip, see fig.1h. It is frequently used in Fracture Mechanics when dimensioning structural members against catastrophic crack growth. The eqs. (1) and (2) may be combined to define a critical value of K_I .

For a moving crack the magnitude of the energy flow into the crack tip varies with the crack propagation velocity $\dot{a}$. Theoretically, the speed at which the newly formed crack surfaces can be separated is limited by the Rayleigh wave velocity C_R in the material which is slightly smaller than the shear wave velocity. For competent granite we have $\dot{a} \approx 1000$ m/s and $C_R \approx 3000$ m/s. Thus, based on a paper by Broberg (1960), an estimate of the magnitude of the dynamic energy flow will yield a value which is approximately 25% lower than the static one and dynamic stress intensity factor values which are approximately 15% lower than the corresponding static ones. In a practical situation the cracks will probably reach an approximately constant propagation speed very fast and then acceleration effects are neglected.

In fact, the difference between the dynamic and the static stress intensity factor values is so small that it seems reasonable to draw conclusions about the radial crack growth from a blast hole from the static ones in the following.

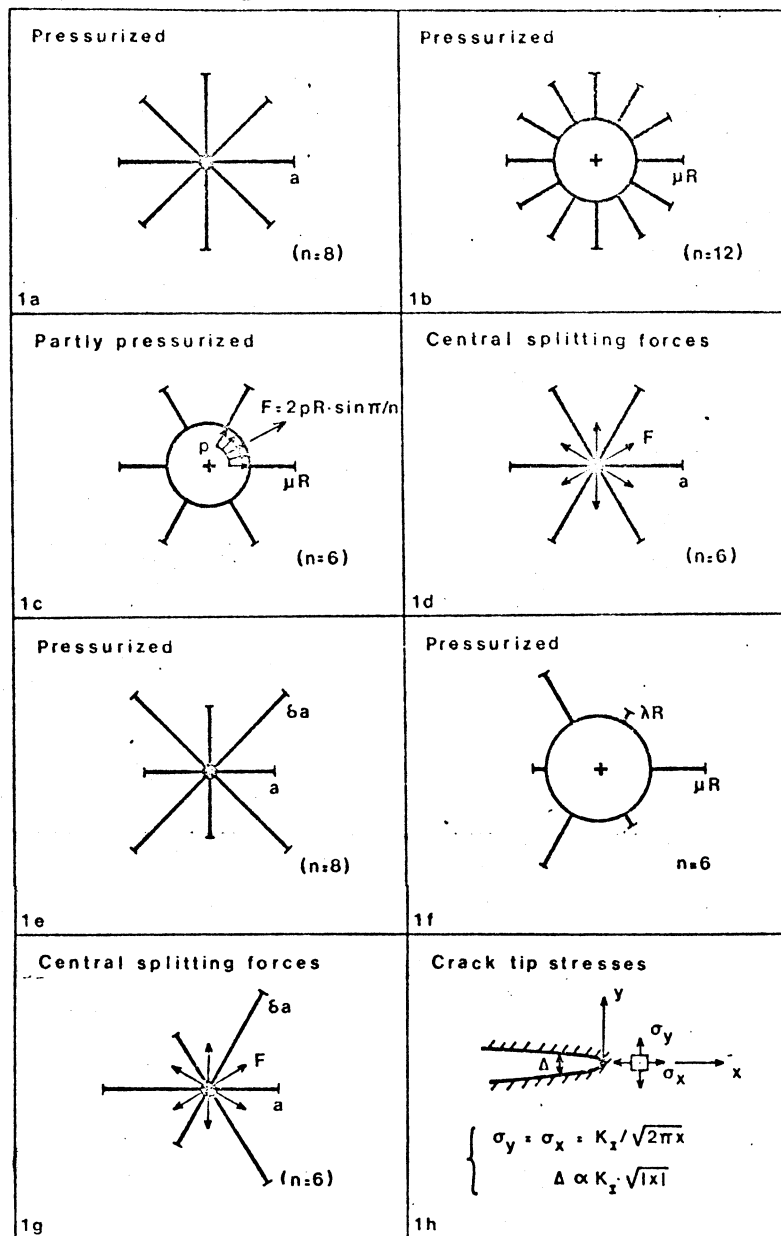


Fig.1. 1a-1h: Load configurations analyzed by Ouchterlony (1972).
1g: General relationship between mode I stress intensity factor and crack tip stresses and tip opening displacement respectively.

RADIAL CRACK GROWTH FROM A BLAST HOLE

The conformal mapping method of Muskhelishvili (1963) has been found to be a powerful tool in the stress analyses of crack systems. Results from Ouchterlony (1972) are presented in diagrams in the following and their relevance to rock blasting is discussed. The most important load configurations are (see fig.1.):

- The pressurized star crack with n arms of equal lengths.
- The pressurized circular hole with n radial cracks of equal lengths.
- The pressurized circular hole with n empty radial cracks of equal lengths.
- Central splitting forces in a star crack with n arms of equal lengths.
- The pressurized star crack with n arms of alternating lengths.
- The pressurized circular hole with n radial cracks of alternating lengths.
- Central splitting forces in a star crack with n arms of alternating lengths.

The configuration in fig.1b was first analyzed by Bowie (1956) for $n = 1$ and 2 cracks and the configuration in fig.1a in turn by Westmann (1964). Kutter (1970) has presented an analysis of both these configurations. Sneddon and Das (1971) have analyzed the configuration in fig.1e for $n = 4$ cracks.

A direct comparison of the various configurations gives information on how the presence of the circular hole influences the crack tip stresses and hence the crack growth. Further, the influence of gas penetration into the cracks and the question of uniform growth of the star crack is treated.

Because of the symmetry of the load configurations the cracks will most likely continue growing radially. Crack branching is not discussed.

The stress intensity factor for the pressurized star crack with n arms of equal lengths is to a high degree of accuracy given by the expression:

$$K_I = 2\sqrt{n-1/n} \cdot p\sqrt{\pi a}, \quad (3)$$

where p is the gas pressure and a is the crack length.

K_I is shown in fig.2. K_I decreases with an increasing number of cracks, asymptotically one has $K_I/p\sqrt{\pi a} \sim 2/\sqrt{n}$. Thus for a given crack length, a star crack with a large number of cracks requires a higher gas pressure to initiate crack growth than does a star crack with a smaller number of cracks. Further, since $K_I \propto (\text{crack length})^{1/2}$ when the gas penetration is complete, the critical gas pressure decreases with increasing size of the star crack.

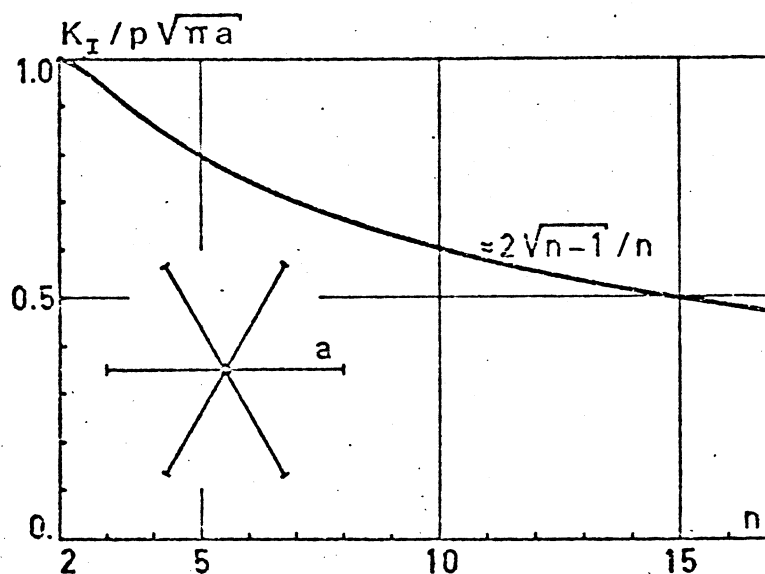


Fig. 2. Normalized stress intensity factor values for the pressurized star crack as a function of the crack number.

The stress intensity factor values for the pressurized circular hole with radial cracks of equal lengths are shown in a set of curves in fig.3. The crack tip to hole radius ratio is denoted μ , see fig.1b. It is observed that, only when the crack tips are close to the hole boundary is the stress field at the crack tips influenced by the presence of the hole. In the limit of very short cracks all the curves tend to the same limiting curve.

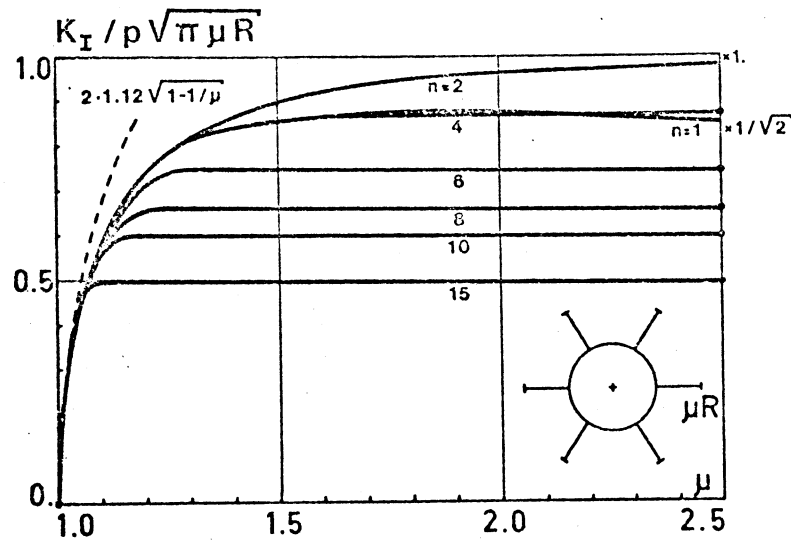


Fig.3. Normalized stress intensity factor values for the pressurized circular hole with radial cracks as a function of crack length for various crack numbers.

A comparison between the pressurized star crack and circular hole configurations is made in fig.4 where the ratio of stress intensity factors circular hole/star crack is plotted. It is seen that the influence of the hole on the crack tip stresses may be neglected with a broad margin when

$$\mu \geq \mu_0(n) = n/(n-2) \text{ for } n > 2, \quad (4)$$

that is whenever the crack length exceeds 1-2 times the hole radius.

Next, let us consider the pressurized circular hole with empty radial cracks depicted in fig.1c. A set of curves is shown in fig.5. One observes that K_I has a maximum for relatively small crack lengths. Thus, with the gas pressure action restricted to the hole boundary, there results a completely different behaviour of stress intensity factor versus crack length than when the gas is able to penetrate into the cracks.

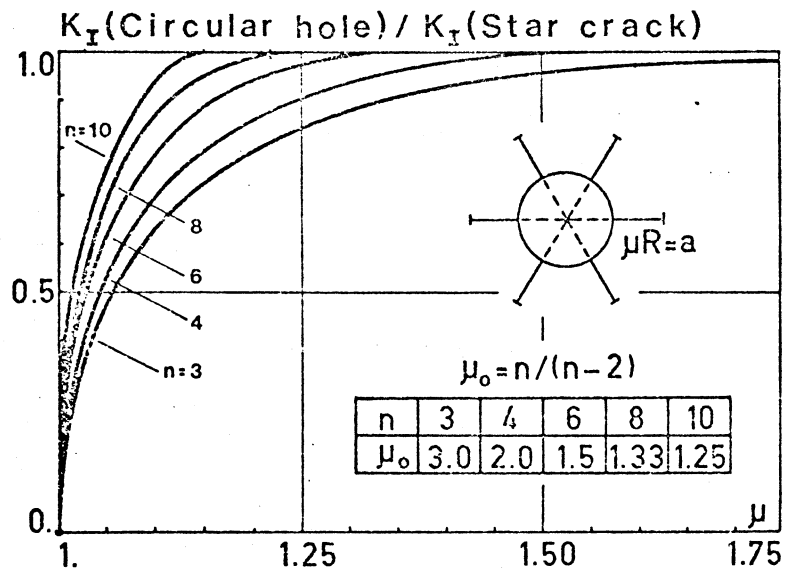


Fig.4. Ratio of stress intensity factors for pressurized contours, circular hole/star crack, as a function of crack length.

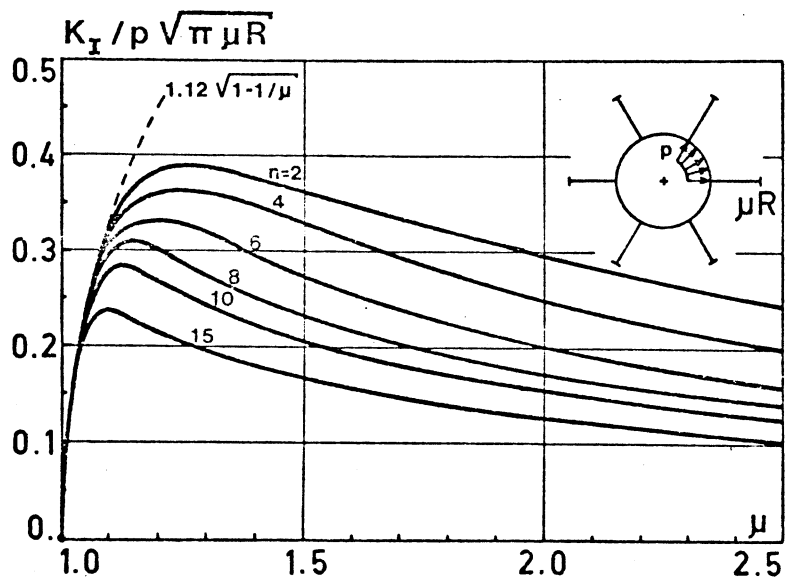


Fig.5. Normalized stress intensity factor values for the pressurized circular hole with empty radial cracks as a function of crack length for various crack numbers.

When the cracks are comparatively long and $\mu \gg 1$ we may replace the gas pressure action on the bore hole sector between two adjacent cracks by it's force resultant $F = 2pR \sin \pi/n$. Then there results, as in fig.1d, the star crack with arms of equal lengths subjected to central splitting forces. The stress intensity factor K_I for this configuration is shown in fig.6. Just as for the pressurized star crack we may separate the shape dependence from the crack length dependence. The result is

$$K_I = k'_I(n) \cdot 2pR/\sqrt{\pi a}. \quad (5)$$

Here the normalized stress intensity factor $k'_I(n)$ depends only on the number of cracks. In the limit of many crack we have $k'_I(n) \approx \pi/2\sqrt{n}$.

Since $K_I \propto (\text{crack length})^{-1/2}$, the critical gas pressure increases with the size of the star crack. This is quite the opposite behaviour to that which is valid for the pressurized star crack. But the conclusion that, for a given crack length, the critical pressure increases with the number of cracks holds true for both configurations.

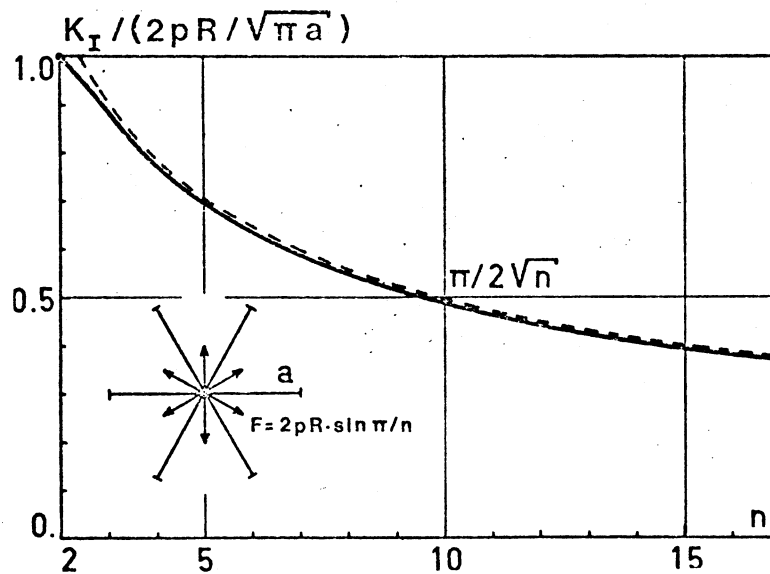


Fig.6. Normalized stress intensity factors for the star crack subject to central splitting forces as a function of crack number.

The influence of the gas penetration on the crack tip stresses is seen in fig.7. Here the stress intensity factor values for the two extremes no gas penetration and total gas penetration into the cracks at the same pressure, in a circular hole with radial cracks, are compared for $n = 6$ cracks. The K_I curves are drawn as solid lines. Shown as a broken line in fig.7 is the asymptotic behaviour corresponding to central splitting forces in the star crack.

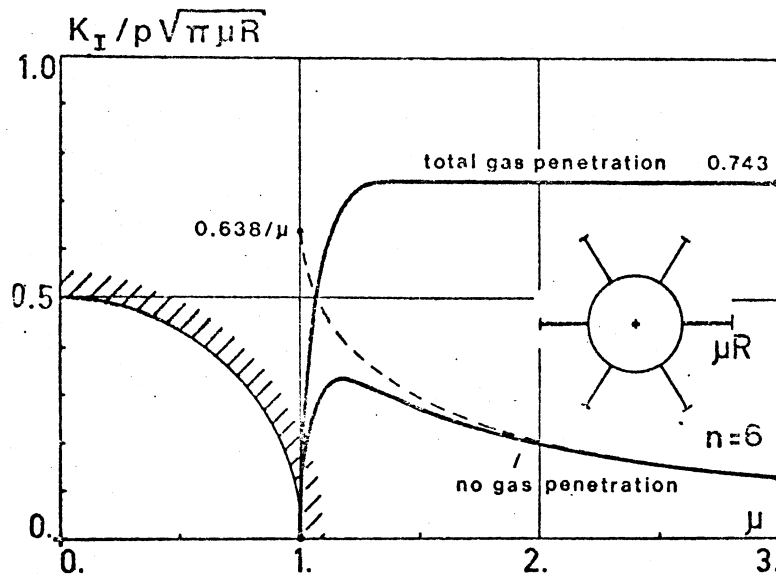


Fig.7. Comparison of normalized stress intensity factor values for the completely and partly pressurized circular hole configurations.

Already for very short cracks the difference between the stress intensity factor values is a factor two. It grows larger with increasing crack length. With eq. (2) one finds that, for a normal blast hole situation, the energy flow into the crack tip at constant gas pressure is a factor $\approx (\pi a / 2R)^2$ larger when the gas is present at the crack tip than it is when the gas pressure action is restricted to the bore hole. For a 1" blast hole with 1 m cracks this gives a factor 10^4 . If the pressure decrease, which is due to the volume expansion of the gaseous detonation products during the blasting process, is taken into consideration one gets a factor 10^2 instead. It is quite obvious, however, that the crack lengths one can get in rock blasting depend heavily on the extent of gas penetration into the crack system.

From fig.7 it is also seen that the crack tip stresses are more sensitive to the presence of the hole when only the hole is pressurized than when the gas penetration is complete. Eq. (4) is then no longer valid. For practical purposes one may neglect the circular hole when the crack length is more than twice the hole radius. If the gas volume expansion has to be considered this might, however, prove a bit hasty.

Now let us turn to those load configurations where the crack arms have different lengths. The first of these is the pressurized star crack with arms of alternating lengths, see fig.1e. The crack length ratio is denoted by δ . We may again extract the shape dependence from the stress intensity factor values. There results

$$\left. \begin{aligned} K_{Ia} &= k_{11}(\delta, n) \cdot p \sqrt{\pi a} \\ K_{I\delta a} &= k_{12}(\delta, n) \cdot p \sqrt{\pi \delta a} \text{ for } 0 < \delta < \infty. \end{aligned} \right\} \quad (6)$$

For reasons of symmetry there follows

$$\left. \begin{aligned} k_{11}(\delta, n) &= k_{12}(1/\delta, n) \\ k_{12}(\delta, n) &= k_{11}(1/\delta, n). \end{aligned} \right\} \quad (7)$$

It is obvious that the values of the stress intensity factors must be the same when the cracks all have the same length. This also follows from eqs. (7) and (6) when $\delta = 1$. The stress intensity factor value is then given by eq. (3).

In fig.8 are plotted pairs of curves for $n = 4, 6, 8, 10$ and 12 cracks. It is observed that, when the crack length ratio δ is either very large or very small, then the shorter cracks exert no influence on the crack tip stresses of the longer ones. This is expressed by: K_{Ia} when $\delta \ll 1$ and $K_{I\delta a}/\sqrt{\delta}$ when $\delta \gg 1$ are both equal to K_I from eq. (3) for $n/2$.

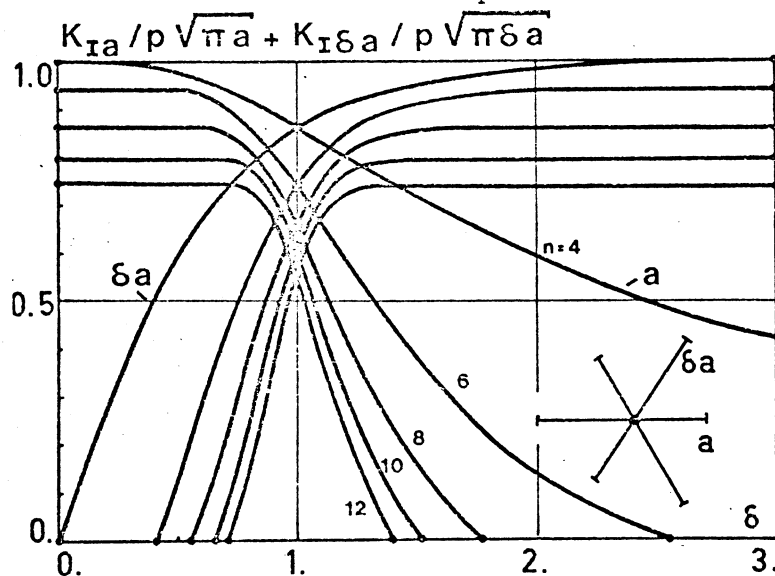


Fig.8. Normalized stress intensity factor values for the pressurized star crack with arms of alternating lengths as a function of the crack length ratio for various crack number.

It is more interesting to note, however, that the curves in fig.8 predict crack tip closure for the shorter cracks. Since the crack tip opening displacement $\Delta \propto K_I$, see fig.1h, tip closure will occur whenever K_{Ia} or $K_{I\delta a}$ tend to become negative. Hence this will occur for $n \geq 6$ cracks when the crack lengths are only moderately unbalanced. The generation of new cracks along with already existing longer ones is also suppressed.

We will now discuss the possibilities for uniform growth of a radial crack system, a matter which has a direct bearing on the crack growth from a blast hole. Since the crack tip stresses are normally not influenced by the hole, this will be considered later as will the effects of gas penetration into the crack system. Also, bearing in mind the estimate of dynamic effects made earlier, no difference will be made between the initiation of crack growth from a static crack system and the continued growth from a dynamic system. The stress intensity factors will be used in the discussion as synonymous of

the energy flow into the crack tips.

The starting point for the discussion is the pressurized star crack with arms of alternating lengths. As is illustrated in fig.9 for $n = 6$ cracks, there follows from fig.8 that $K_{Ia} > K_{I\delta a}$ for $\delta < 1$ and conversely for $\delta > 1$ when $n \geq 4$ cracks. Consequently, an initiation of crack growth will only involve the longer cracks. Thus a uniform growth of the configuration is impossible even if all the cracks initially were to have the same length. Since in reality neither the load configuration nor the material conform exactly to the ideal, a small initial imbalance will always be amplified. This predominance of the longer cracks over the short ones is perhaps intuitively accepted because both the general crack length dependence of the stress intensity factors, $K_I \propto (\text{crack length})^{1/2}$, and the relative clamping of the short cracks point in this direction. The clamping action may be thought of as resulting from the compressive stresses induced in a sector by the gas pressure in the two adjacent longer cracks. Both the clamping and the unbalanced growth become more pronounced as the number of cracks increases. This may be seen in fig.8. For the pressurized plane crack when $n = 2$, however, neither crack tip is more likely to start growing.

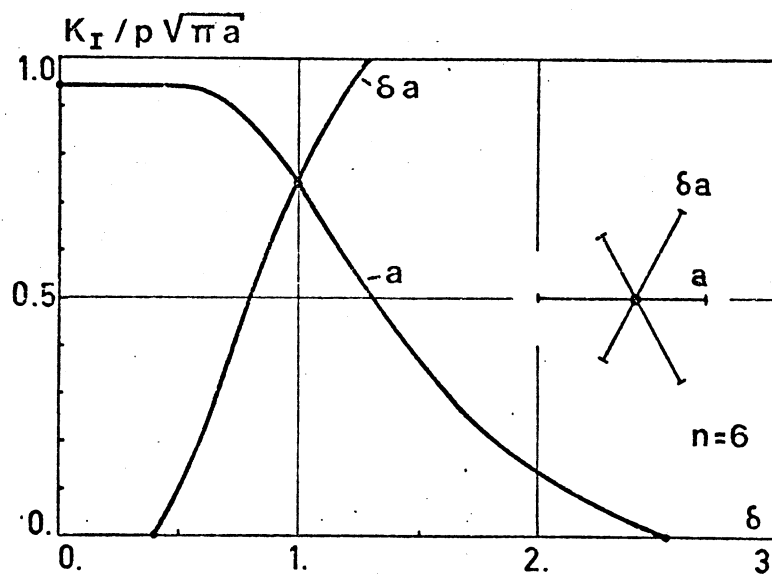


Fig.9. Comparison of normalized stress intensity factor values for the different length crack arms of a pressurized star crack with 6 arms of alternating lengths.

The conclusions which have been drawn for the pressurized star crack with arms of alternating lengths remain essentially unchanged when the effects of the hole are considered as in fig.10 for $n = 6$ cracks. Here μ and λ denote the occurring crack tip distance to hole radius ratios, see also fig.1f. A direct comparison with the star crack values is obtained if one sets $\delta = \lambda/\mu$ and $a = \mu R$ as depicted by the dashed lines. It is seen

that the influence of the hole on the crack tip stresses is negligible when both λ and $\mu \geq \mu_0$ ($n=6$) = 1.5. This is true for any number of cracks, such that $n \geq 4$.

The complete results for the pressurized circular hole with radial cracks of alternating lengths indicate that crack tip closure will not occur very close to the hole boundary. But the generation of new cracks from the hole is suppressed which means that the effect is essentially the same as closure. This follows from the observation that K_I tends to zero with zero crack length.

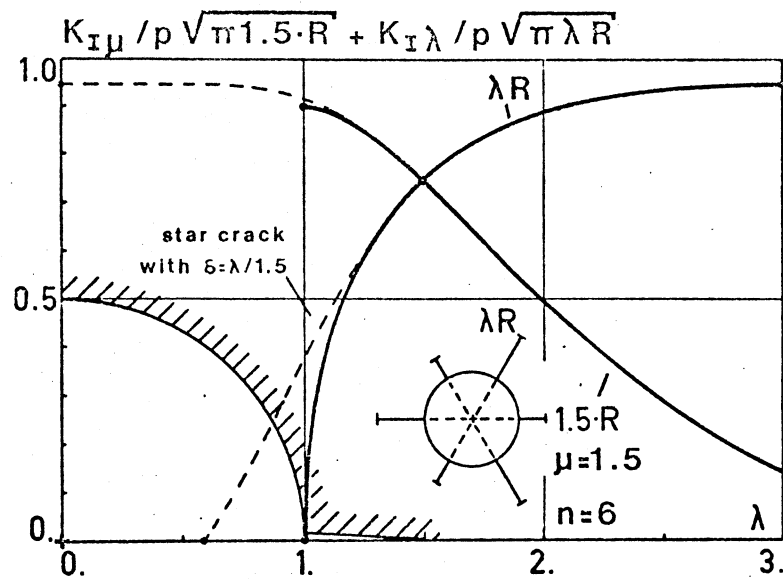


Fig.10. Comparison of normalized stress intensity factor values for pressurized star crack and circular hole with radial cracks respectively both with 6 arms of alternating lengths.

Finally, in order to assess the influence on the radial crack growth of the gas penetration into the crack system, the results for the star crack with arms of alternating lengths subjected to central splitting forces in fig. 1g are presented in fig.11. As for the pressurized contour the shape dependence is extracted from the stress intensity factors by writing

$$\left. \begin{aligned} K_{Ia} &= k_{11}'(\delta, n) \cdot (2pR/\sqrt{\pi a}) \\ K_{I\delta a} &= k_{12}'(\delta, n) \cdot (2pR/\sqrt{\pi \delta a}). \end{aligned} \right\} \quad (8)$$

Here $k_{11}'(\delta, n)$ and $k_{12}'(\delta, n)$ satisfy the symmetry relations in eq. (7).

The crack growth behaviour of this load configuration is more complicated than that of the pressurized contour. The general crack length dependence of the stress intensity factors, $K_I a(\text{crack length})^{-1/2}$, works in favor of an initiation of crack growth from the shorter crack tips but this is counteracted by the clamping action. A careful analysis of the results will reveal that

the general crack length dependence dominates only when the number of cracks is small.

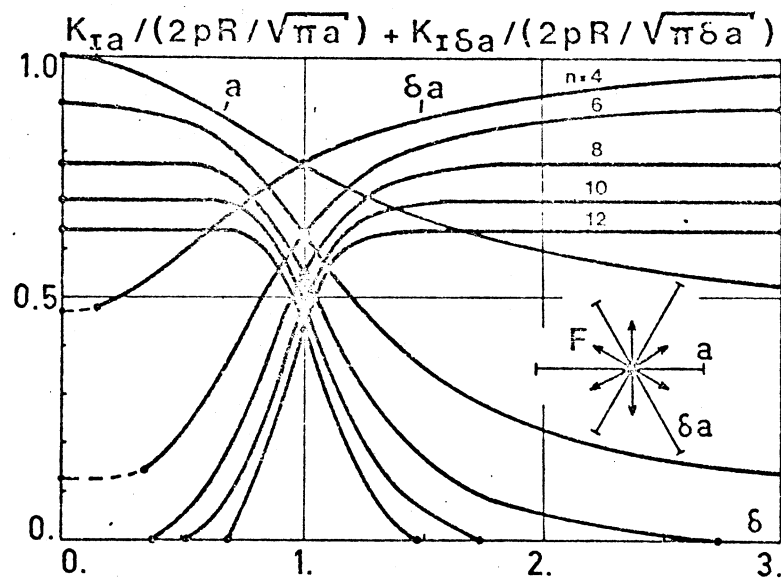


Fig.11. Normalized stress intensity factor values for the star crack with arms of alternating lengths and central splitting forces as a function of the crack length ratio for various crack numbers.

The stress intensity factors for the plane crack subjected to off-center splitting forces are easily derived in the form

$$\left. \begin{aligned} K_{Ia} &= \sqrt{2\delta/(1+\delta)} \cdot (2pR/\sqrt{\pi a}) \\ K_{I\delta a} &= \sqrt{2/(1+\delta)} \cdot (2pR/\sqrt{\pi \delta a}). \end{aligned} \right\} \quad (9)$$

Since $K_{Ia} < K_{I\delta a}$ when $\delta < 1$ and vice versa when $\delta > 1$, the initiation of crack growth will always start from the tip of shorter crack. Eventually a situation arises where the plane crack with central splitting forces grows uniformly.

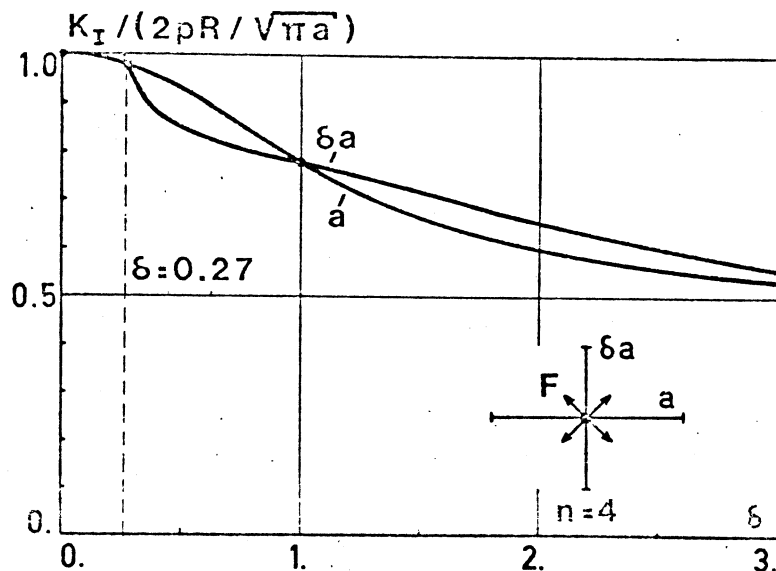


Fig.12. Comparison of normalized stress intensity factor values for the different length crack arms of a star crack with 4 arms of alternating lengths subject to central splitting forces.

The curves in fig. 12 demonstrate that this pattern of behaviour is changed when $n = 4$. When the crack length ratio lies within the interval $0.27 < \delta < 3.7$, crack growth will initiate from the tips of the longer cracks otherwise from the tips of the shorter ones. Thus any preferential growth will be stopped when δ becomes either 0.27 or 3.7. From then on a uniform growth of the star crack with arms of alternating lengths is possible. This behaviour is probably also relevant for $n = 6$ cracks, but one notes a widening of the instability interval. For $n \geq 8$ cracks this instability is fully developed and a uniform growth of the star crack with arms of alternating lengths subjected to central splitting forces is no longer possible. As was the case for the pressurized contour this also involves crack tip closure. It is a reasonable surmise that the effect of the circular hole again will be to suppress the generation of new cracks from the hole and to check any preferential growth of already existing short cracks.

To wind it all up let us return to the rock blasting process, described by for example Persson and co-workers (1970). Shortly after the detonation of the charge a great number of small cracks are generated at the hole boundary. The foregoing discussion has demonstrated that a radial crack system has an inherent tendency towards non-uniform growth. The detailed reasoning shows that during the radial growth, cracks will successively fall behind the crack front and their further growth be suppressed. This provides a qualitative basis for understanding why only a few of the original cracks will grow to a substantial length. It is evident from the discussion that this mechanism is largely insensitive to gas penetration into the crack system.

According to Persson and co-workers (1970), the number of the dominating cracks is often around five. It may not be pure coincidence that this is in the interval $n = 4$ to 6 where the growth pattern of a radial crack system with no gas penetration into the cracks changes from non-uniform to uniform.

CONCLUSIONS

An application of Fracture Mechanics to rock blasting or more specifically to the radial crack growth from a blast hole, gives a physically sound qualitative understanding of this process in which the cracks are explicitly considered. In summing up, one may conclude that for a radial crack system there follows, irrespective of the gas penetration, that:

- a. The critical gas pressure for crack growth increases with an increasing number of cracks.
- b. The circular hole no longer influences the crack tip stresses outside a near-zone of 2-3 hole radii.
- c. The presence of short cracks doesn't influence the crack tip stresses of longer ones when the crack length ratio is larger than 2-3.
- d. In the presence of long cracks, tips of shorter cracks tend to close.

- e. A uniform growth of all cracks in a radial crack system is virtually impossible. This imbalance becomes more accentuated with an increasing number of cracks.

However:

- f. The crack lengths one may obtain in rock blasting are strongly influenced by the extent of the gas penetration.

Fracture Mechanics is also expected to be a useful tool when analysing other parts of the fragmentation process in rock blasting. The validity of the conclusions presented here is naturally not restricted to rock blasting. The hydraulic fracturing of drill holes should for example prove one field of application.

ACKNOWLEDGEMENT

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