

PREDICTION OF CRACK LENGTHS IN ROCK AFTER CAUTIOUS BLASTING WITH ZERO INTER-HOLE DELAY

**Beräkning av spricklängder i kvarstående berg
efter skonsam sprängning
med momentan upptändning av flera hål**

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FÖRORD

SveBeFo:s forskningsprogram spänner över bergbyggandets olika stadier, från undermarksplanering och förundersökningar till berggutttag, förstärkning, tätning och långtidsbeständighet hos anläggningar i drift. Forskningen bedrivs till stor del med externa forskare vid högskolor och företag, medan specialområdet sprängningsteknik och sprängämnesforskning huvudsakligen genomförs med egna forskare, anställda vid SveBeFo.

Inom sprängningstekniken är en huvuduppgift att minska eller helt eliminera sprängskador och sprickbildning utanför den sprängda konturen i kvarstående berg. Sedan flera år pågår fältförsök med skonsam sprängning i ett stenbrott i Vånga norr om Kristianstad där en mängd parametrar varierats och många intressanta resultat erhållits. Bland annat har betydelsen av momentan upptändning av konturhålen för att minimera sprickdjupet påvisats liksom t.ex. frikopplingens stora betydelse för erhållna spricklängder.

Traditionella formler för beräkning av sprängskador tar inte hänsyn till detta. Parallellt med Vångaförsöken har därför ett arbete bedrivits för att utveckla bättre sprängskad modeller så att inverkan av faktorer som frikoppling, detonationshastighet, spännings effekter mm på sprickbildningen kan förutsägas. Föreliggande rapport beskriver härledningen av nya prognosekvationer för radiell sprickbildning vilka baseras på försöksresultaten från Vånga.

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Finn Ouchterlony

Sammanfattning

I denna rapport används mekanikens lagar och dimensionsanalys för att härleda nya prognosekvationer för längden R_c hos radiella sprickor som bildas i bergkonturen bakom spränghål. Det fall som behandlas är skonsam sprängning med fri botten och utan fördröjning mellan de olika spränghålens upptändning. De parametrar som ingår i ekvationerna är;

- densitet ρ_e , detonationshastighet D och adiabatisk expansionsexponent γ för sprängämnet,
- borrhålsdiameter $\emptyset_h$ och laddningens kopplingsgrad f , samt
- densitet ρ , ljudhastighet c och brottseghet K_{lc} hos berget.

Den presenterade teorin förutsäger att det kritiska trycket $p_{h,crit}$ för sprickinitiering vid dynamisk trycksättning av borrhål beror på hålstorleken enligt $p_{h,crit} = 1,52 \cdot K_{lc} / \sqrt{\emptyset_h}$. Försöksdata beskrivs emellertid bäst av koefficienten 3,30, se nedan.

Ett antal parametrar i prognosekvationerna har bestämts genom kurvanpassning till försöksdata från pallsprängning i ett stenbrott i Vånga i Skåne. Data motiverar bara användning av en förenklad form hos ekvationerna. En kompromiss mellan ekvationernas komplexitet och anpassningens godhet ges av

$$p_h = \gamma^{\gamma/(\gamma+1)} \cdot \rho_e D^2 f^{2,2}, \quad (43a)$$

$$p_{h,crack} = 3,30 \cdot K_{lc} / \sqrt{\emptyset_h}, \text{ och} \quad (43c)$$

$$2R_c / \emptyset_h = (p_h / p_{h,crack})^{2/[3(D/c)^{0,25}-1]}. \quad (43b'')$$

Här är $p_{h,crack}$ det kritiska trycket för dynamisk spräckning av borrhålet som analysen av fältdata gett. Korrelationskoefficienten är 0,98.

Ett konstruktionsdiagram ges, baserat på dessa ekvationer och användningen förklaras. Det ger en helhetsbild av hur ingående faktorer påverkar beräknade spricklängder. Slutmålet för arbetet är att bidra till ett bättre underlag för beräkning av skonsam sprängning men de prognosverktyg som hittills tagits fram måste användas med stor försiktighet på grund av de begränsningar som analysen och underliggande data har.

En jämförelse med SveBeFo's egna uppmätta fältdata visar att prognosekvationerna fungerar väl för de två sprängämnena med låg detonationshastighet, VOD eller D , som använts. Det är Gurit och Emulet 20 för vilka $D \approx 2000$ m/s. Resultatet är sämre för de

två sprängämnen med hög VOD, Kimulux 42 och Detonex 80. För Kimulux överskattas spricklängden med ca 70% och för Detonex underskattas den med ca 25%. En jämförelse med fristående data bekräftar att prognosekvationerna fungerar för två andra sprängämnen med låg VOD och att de spricklängder som Detonex ger underskattas. Detta och andras arbeten pekar på svagheter i beskrivningen av sprängämnenas egenskaper som borde rättas till.

Prognosekvationerna ovan ger inget rent beroende av laddningskoncentrationen q (kg/m). För sprängämne med hög detonationshastighet, $D \approx 5500$ m/s, respektive låg, $D \approx 2000$ m/s, reduceras de till

$$R_c \approx \text{konst} \cdot q \cdot e / \bar{O}_e^{0,5} \cdot f^{0,7} \text{ när } D = c, \text{ och} \quad (46a)$$

$$R_c \approx \text{konst} \cdot (q \cdot e)^{1,5} / \bar{O}_e^{1,25} \cdot f^{1,55} \text{ när } D = 0,36 \cdot c. \quad (46b)$$

Här betecknar e (J/kg) sprängämnets explosionsenergi. För ett givet sprängämne är nettoeffekten att $R_c \approx \text{konst} \cdot q^{0,75-0,88}$ vilket är mindre än det traditionella lineära q -beroendet. Å andra sidan verkar det som om sprängämnen med låg VOD skulle vara känsligare för en frikoppling än sprängämnen med hög VOD, något som inte är självklart.

Till slut ges några förslag på hur prognosekvationernas giltighet skulle kunna förbättras och utvidgas genom ytterligare försök och en bättre beskrivning av sprängämnenas detonationsprodukter. Denna rapport skall ses som första steget mot en tillfredställande ingensjörsmässig beräkning av spricklängder i kvarstående berg efter skonsam sprängning.

Det bör därför påpekas att här presenterade ekvationer och konstruktionsdiagram baseras på en analys med sådana begränsningar att varje spricklängdsprognos för en verklig sprängsalva i praktiken måste användas med stor försiktighet.

Nyckelord: Skonsam sprängning, momentan upptändning, sprängskador, prognoser, radiella sprickor, spricklängd, frikoppling, sprängämnen, VOD, detonationshastighet, brottseghet.

Summary

In this report, mechanics and dimensional analysis are used to derive new prediction equations for the lengths R_c of the radial cracks emanating from the half-casts in the contour after cautious blasting of a contour with a free toe and zero initiation delay between the charges. The influencing parameters of the equations are;

- density ρ_e , velocity of detonation D and adiabatic expansion exponent γ of the explosive,
- bore-hole diameter $\varnothing_h$ and charge coupling ratio f , and
- density ρ , sound velocity c and fracture toughness K_{Ic} of the rock.

The theory presented predicts, among other things, that the critical fracturing pressure $p_{h,crit}$ for a dynamic pressurization of a bore-hole depends on hole size, $p_{h,crit} = 1,52 \cdot K_{Ic} / \sqrt{\varnothing_h}$. The experimental data is however best described by the prefactor 3,30, see below.

A number of parameters in the prediction equations are determined by curve fitting to test data obtained from bench blasting in granite in the Vånga quarry. These data do not motivate the use of the full form of the equations however. A compromise between equation complexity and goodness of fit is obtained by

$$p_h = \gamma / (\gamma + 1)^{(\gamma + 1)} \cdot \rho_e D^2 f^{2,2}, \quad (43a)$$

$$p_{h,crack} = 3,30 \cdot K_{Ic} / \sqrt{\varnothing_h}, \text{ and} \quad (43c)$$

$$2R_c / \varnothing_h = (p_h / p_{h,crack})^{2/[3(D/c)^{0,25-1}]}. \quad (43b")$$

Here $p_{h,crack}$ is the critical pressure level for fracturing of the bore-hole that follows from the field data. The coefficient of determination for these equations is 0,97.

An approximate design diagram for these equations is given and its use explained. This diagram gives a comprehensive view of how the different parameters influence the predicted crack lengths. The end goal of this work is to help provide a basis for improved design of cautious blasting but the prediction tools of this report must be used with great caution because of the limitations of both the analysis and of the underlying data.

A comparison with SveBeFo's Vånga data shows that the prediction equations are very good for the two low VOD explosives used, Gurit and Emulet 20 for which $D \approx 2000$ m/s. It is not that good for the two high VOD explosives used, Kimulux 42 and Detonex 80.

The crack lengths generated by Kimulux are overestimated by about 70% and those caused by Detonex underestimated by about 25%. A comparison with independent data confirms the prediction quality for two other low VOD explosives and that the crack lengths caused by Detonex are underestimated. This and other referenced work point to some deficiencies in the explosives description which should be considered.

A pure dependence on charge concentration q (kg/m) is not inherent in the prediction equations obtained here. For a high VOD explosive, $D \approx 5500$ m/s, and a low VOD explosive, $D \approx 2000$ m/s, respectively they reduce to

$$R_c \approx \text{const} \cdot q \cdot e / \varnothing_e^{0,5} \cdot f^{0,7} \text{ for } D = c, \text{ and} \quad (46a)$$

$$R_c \approx \text{const} \cdot (q \cdot e)^{1,5} / \varnothing_e^{1,25} \cdot f^{1,55} \text{ for } D = 0,36 \cdot c. \quad (46b)$$

Here e (J/kg) denotes the specific energy of the explosive. For a given explosive the net effect is that $R_c \approx \text{const} \cdot q^{0,75-0,88}$ which is less than the commonly used linear relationship. On the other hand a low VOD explosive would seem to be relatively more sensitive to decoupling than high VOD one, an observation which not self-evident.

At the end, tests are recommended and explosives descriptions suggested that could improve the predictive capacity of the equations presented here. This report is a first step towards a satisfactory engineering prediction of crack lengths in rock after cautious blasting.

It should thus be repeated that the equations and design diagram presented here are based on an analysis with such limitations that any crack length predictions that can be made for a real, practical blast round must be used with great caution.

Key words: Cautious blasting, zero delay, blast damage, prediction equation, radial cracks, crack length, decoupling, explosives, VOD, fracture toughness.

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1 INTRODUCTION

To be able to design blasting rounds so that the damage to the remaining rock doesn't create serious problems has always been of prime importance in rock excavation. In tunnels and drifts, the extra costs of cautious contour blasting can be recovered by less scaling and support work or less concrete backfill (Andersson, 1994). In open pit mining, the cautious blasting leads to improved safety from better functioning catchment berms and improved recovery of ore through steeper slopes (Siira, 1994).

The term damage should be reserved for the degree of impaired function of the structure, e.g. its stability. It is however often used as synonymous with cracking; micro- or macrocracking, even movement in and extension of existing cracks. The design of cautious blasting is conceptually based on the cracking but often expressed in terms of peak particle velocity, PPV (Holmberg and Persson, 1980; Hustrulid et al., 1992). These designs mainly serve as a consistent way to keep down the charge concentration (Niklasson et al., 1994), i.e. the dynamic load level, and the resulting damage is seldom documented except by the superficial half-cast factor measure (Ouchterlony, 1992).

Even in extensive blast damage monitoring studies (Ouchterlony et al., 1993; Olsson, 1996), the main thrust in the damage measurements lies in indirect methods, geophysical, hydrological etc. Recently SveBeFo has developed a direct technique to measure the extent of cracking behind blast-holes and conducted an extensive series of tests under controlled conditions in a dimensional stone quarry at Vånga (Olsson and Bergqvist, 1993a-b and 1996a-b). The purpose of this work has been to obtain a basis for improved contour blasting of mainly tunnels.

At Vånga, 4-5 m long, Ø 24-64 mm blast-holes in various burden-spacing patterns have been charged with different explosives and blasted in groups of three or four along benches. To date over 300 holes have been fired since 1991, most of them with zero inter-hole delay, using electronic delay detonators. The remaining damaged contour has been carefully removed by a cord post-split and the resulting blocks wedge split to manageable size. The blocks have then been sawn perpendicularly to the half-casts and the mainly radial cracking visualized using dye penetrant techniques. For an example of an individually fired hole see Figure 1 where the radial cracks clearly dominate.

The results obtained so far at Vånga may be summarized as follows. For otherwise constant conditions:

- The crack lengths decrease with a decreasing coupling ratio.
- Fully charged holes show a more complex crack pattern with interacting cracks.

- A high velocity of detonation, VOD, results in many short cracks around the hole.
- The crack lengths increase with an increasing charge concentration.
- For some explosives the crack lengths increase with an increasing burden.
- Arc-shaped subsurface cracks may occur when the spacing is increased.
- The time delay between initiation of the different holes is important;
 - an instantaneous initiation of groups of holes gives the shortest cracks,
 - the crack lengths increase with an increasing delay time,
 - even delay times as low as 1 ms give crack lengths more like single hole blasts,
 - sequential microinterval (zipper) blasting is inferior to instantaneous initiation,
 - traditional smooth blasting procedures give unnecessarily long cracks and
 - alternating initiation sequences may give short cracks.

The Vånga work is expected to continue for at least the next 3 years.



Figure 1: Radial cracks behind 51 mm blast-hole, Olsson and Bergqvist (1993a).

In order to make the best use of the unique Vånga data base, it should be used to verify/falsify existing models for cracking or to develop new ones. Such models can then be used to design cautious blasting with increased confidence. Such work started within SveBeFo's 1993-96 research program and this is a report of the first results from a quasistatic modelling based on mechanics and dimensional analysis.

Table 1: List of symbols used.

a	= acceleration, m/s^2	$p_{h,crack}$	= experimental value of $p_{h,crit}$
a	= exponent in curve fit eqn. 42	p_s	= shock pressure amplitude, Pa
B	= burden, m	$p_{s,crit}$	= critical shock amplitude, Pa
B_c	= critical burden, m	PPV	= peak particle velocity, m/s
c	= sound velocity in rock, m/s	PPV_o	= fully coupled PPV, m/s
d	= uncharged hole length, m	PPV_c	= critical PPV, m/s
D	= velocity of detonation, m/s	q	= charge concentration, kg/m
e	= explosive energy, J/kg	r	= distance from hole center, m
e, e'	= exponents in curve fit eqn. 42	r^2	= coefficient of determination
f	= coupling ratio	R_c	= max. length of radial cracks, m
$f(R/H)$	= charge length correction, eqn. 7	s	= Hugoniot slope
E	= Young's modulus of rock, Pa	S	= burden, m
F	= force, N	Δt	= initiation delay, s
g_o	= conversion constant, $\text{kg}\cdot\text{m/s}^2/\text{N}$	u_r	= radial displacement, m
H	= height of explosive column, m	$\dot{u}_r$	= radial particle velocity, m/s
i	= impedance function in eqn. 42	$\dot{u}_s$	= shock velocity, m/s
l	= rock damping coefficient, 1/m	v	= crack velocity, m/s
k	= pressure prefactor, $\text{cm}/\text{MPa}^{0.68}$	α	= decoupling exponent
K_{la}	= crack arrest toughness, $\text{Pa}\sqrt{\text{m}}$	β	= pressure decay exponent
K_{lc}	= fracture toughness, $\text{Pa}\sqrt{\text{m}}$	δ	= pressure decay exponent
m	= mass, kg	ε_θ	= tangential or hoop strain
n	= pressure prefactor	$\dot{\varepsilon}_\theta$	= hoop strain rate, 1/s
N	= number of radial cracks	γ	= adiabatic exponent
p	= pressure, Pa	λ	= pressure prefactor
p_{CJ}	= CJ-point pressure, Pa	$\emptyset_e$	= charge diameter, m
p_e	= constant volume explosion pressure, Pa	$\emptyset_h$	= bore-hole diameter, m
p_h	= bore-hole pressure, Pa	ν	= Poisson's ratio of rock
$p_{h,crit}$	= critical bore-hole pressure, Pa	ρ	= rock density, kg/m^3
		ρ_e	= explosive density, kg/m^3

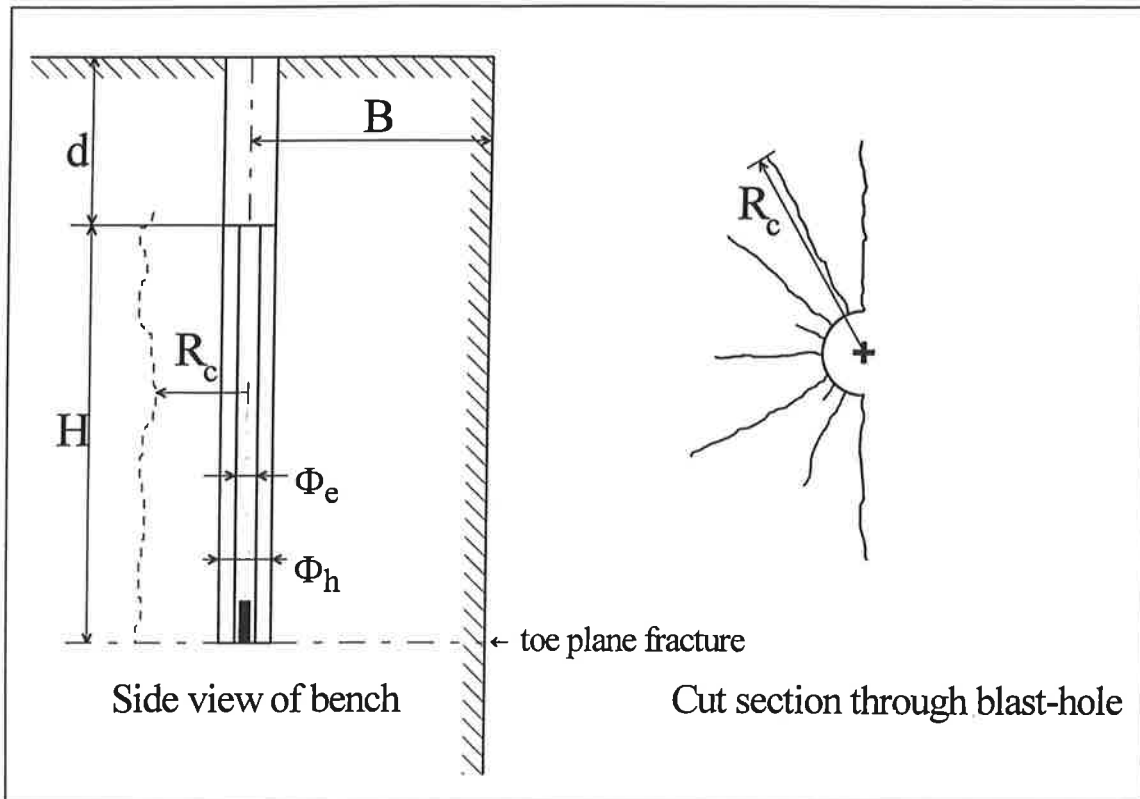


Figure 2: Side view of cautious blasting with a free bottom.

2 DERIVATION OF MODEL

2.1 Description of process and general formula

A simplified picture of the Vånga bench blasts is shown in Figure 2. At Vånga there is a horizontal fracture plane which cuts off the toe and no subdrilling or stemming has been used. The primer has been minimal, 50 g, so the situation is as nearly two-dimensional as one can hope to get in practice. In most cases, the charges have been radially decoupled, i.e. their diameter smaller than the blast-hole diameter.

A very brief description of the sequence of events of the blast and the main influencing factors may then be given as follows. The nomenclature is given in Table 1 above.

- | | |
|-------------------------------------|------------------------------|
| 1. Detonation of charge | $\rho_e, D, \gamma, \rho, c$ |
| 2. Gas pressure hits bore-hole wall | $\emptyset_e, \emptyset_h$ |
| 3. Crack growth is initiated | E, ν, σ_t, K_{Ic} |

4. "Symmetric" growth of cracks	N, p_h
5a. Stress wave reflection from free face	B, d, c
5b. Shock wave from adjacent charge	$S, \Delta t$
6. Preferred growth of cracks	B
7a. Cracks vent to free face	v, B
7b. Axial venting of gases	stemming, d, H
8a. Burden starts moving out	$B < B_c$
8b. Contour cracks stop moving	K_{la}, R_c

Note that each event is not necessarily well defined. Nor is the time order necessarily always correct as indicated by 5a-5b etc. The list is meant to give a qualitatively correct idea of the parameters involved in the blasting process.

If we regroup the influencing parameters in a dimensional analysis scheme (Taylor, 1974) we obtain

Explosive loading:	4	$\rho_e, D, \gamma, \Delta t$
Geometry:	6	$\emptyset_e, \emptyset_h, B, S, H, d$
Rock material:	7	$E, \rho, c, v, \sigma_t, K_{lc}, K_{la}$
Result parameter:	1	R_c or N
Physical dimensions:	4	$F(N), M(kg), L(m)$ and $T(s)$
Newton's second law:	1	$F = ma/g_o, g_o(kg \cdot m/s^2/N).$

This means that the minimum number of dimensionless groups to describe the process is thus $4+6+7+1-4+1 = 15$. This number is much too large to be practicable. The first possible reduction stems from a relationship between the rock material parameters, see Achenbach (1973) e.g.

$$E/g_o = \rho c^2 \cdot g(v), \quad (1)$$

where $g(v)$ is a dimensionless function of the Poisson's ratio v which depends on which one of the elastic wave propagation velocities that c represents. In the SI system g_o assumes the value 1. For simplicity we assume that c is the P-wave velocity and then

$$g(v) = \sqrt{[(1-v)/(1+v)(1-2v)]}. \quad (2)$$

If $v \leq 0,25$, which is realistic for rock, then $g \leq 1,1$. We shall thus assume that the influence of v is insignificant and use $g \approx 1$. This is tantamount to saying that we can use the bar velocity instead of the P-wave velocity and that the parameters v and e.g. E drop out of the dimensional analysis scheme above.

By equations 1 and 2, the number of dimensionless groups is reduced by 2 to 13. To obtain a further reduction we shall assume that the process is two-dimensional. This removes the parameters H and d . Thus the number of groups becomes 11.

Then we further restrict the analysis to a constant blasting pattern, $S/B \approx \text{constant}$, a single hole or instantaneous initiation of all holes, $\Delta t = 0$, and one strength parameter for the rock. For example, σ_t could essentially be derived from K_{Ic} or at least we assume that intact strength behavior is less important to our process than crack propagation and arrest. The latter phenomenon is for simplicity assumed to be governed by $K_{Ic} \approx K_{Ia}$.

Then 7 groups remain and a general functional relationship can be stated as e.g.

$$R_c/\emptyset_h = h(\rho_e \emptyset_e^2 / \rho \emptyset_h^2, \emptyset_e/\emptyset_h, B/\emptyset_h, c/D, \rho_e D^2 \sqrt{\emptyset_h/K_{Ic}}, \gamma). \quad (3)$$

In this equation the groups may be rearranged to suit different purposes as long as all the remaining 10 parameters appear at least once. Other intermediate or derived parameters may be replaced by the original ones on a one to one basis.

We know that the charge concentration $q(\text{kg/m})$ may be written

$$q = \pi/4 \cdot \rho_e \emptyset_e^2, \quad (4)$$

and that the coupling ratio f is defined by

$$f = \emptyset_e/\emptyset_h. \quad (5)$$

Thus, provided that the function $h(\dots)$ is known, equation 3 answers the question "Given q , f , D , and B etc, how long will the (maximum) crack length in a given rock be?". These are exactly the variables involved in the Vånga work, see the Vånga result summary above. Hence these results could be used to determine h . We have no a priori knowledge of its functional form though. Thus some simple approaches will be tried.

2.2 Some simple formulas

As a Swedish rule of thumb, $R_c(m) = q(\text{kg/m of Dynamex M equivalents})$ was often used before. Work by Sjöberg (1979) replaced this equation by

$$R_c = 1,9 \cdot q(\text{kg/m of Dynamex B equivalents}) \text{ when } q < 1,4. \quad (6)$$

For each explosive, the equivalent amount of Dynamex B must be calculated using the weight strength concept (Persson et al., 1993). This equation is the origin of several tables that have been in extensive use. Since it relates the damage zone depth only to the charge concentration it can not account for the sizeable effect of decoupling that we have seen in the Vånga tests. Thus it is insufficient for our modelling purposes.

In the engineering approach of Holmberg and Persson (1980), a critical PPV-value is used to delimit the damage zone. Often $PPV_c = 700\text{-}900 \text{ mm/s}$. As simplified by Ouchterlony et al. (1993), the PPV-value at a distance R may be calculated from the site scaling law,

$$PPV = A \cdot [\sqrt{(fqH)/R}]^\beta \text{ with } f = \tan(H/2R)/(H/2R). \quad (7)$$

Here A and β are site specific constants, determined by measurements. The function $f(R/H)$ is a correction for charge length H , based on the observation that close to the charge, only parts of the charge seem to contribute to the PPV value. When R/H becomes large, $f \rightarrow 1$.

The calculation of R_c from equation 7 was formulated as follows,

$$(PPV_c/A)^{1/\beta} / \sqrt{(q/H)} \cdot R_c/H = \sqrt{f(R_c/H)}, \quad (8)$$

which has a simple graphical solution. Ouchterlony et al. (1993) found a good correlation between this equation and geophysical measurements of the damage zone around a tunnel, blasted with different explosives.

Again, because the decoupling is missing, this equation is unsuitable in its present form. The effect decoupling may of course be added, see Atchison et al. (1964) or Atchison (1968) who have found that, relative to the fully coupled state PPV_o , the decoupled PPV value is given by

$$PPV = PPV_o \cdot (\emptyset_e / \emptyset_h)^{1,5} = PPV_o \cdot f^{1,5}. \quad (9)$$

A commonly used value for the decay exponent in equation 7 is $\beta \approx 1,5$. In cautious blasting shallow damage zones are in focus, $R_c \ll H$. Then equations 7-9 reduce to

$$R_c = \pi(PPV_c/A)^{4/3} \cdot q \cdot f^2. \quad (10)$$

Still, other explosive properties such as VOD are poorly represented in these equations.

Recently Hustrulid et al. (1992, 1994) have presented another PPV formula for blast damage zone predictions. It may be written

$$PPV = 0,61 p_r / (\rho c) \cdot (\emptyset/R) \cdot \exp(-0,61 l R / \emptyset). \quad (11)$$

Here p_h is the pressure on the bore-hole wall and l is a rock damping factor. In Hustrulid (1994), the $\emptyset$ -value in the exponent is included in the damping factor of dimension 1/m. Equation 11 is based on a linearly elastic solution for the PPV from a spherical source and the theoretical finding that the effect of a cylindrical charge is given by the equivalent sphere of height $b = \sqrt{1,5} \cdot \emptyset / 2$ which is closest to the gauge point. Normally multiple spheres need not be considered so the charge length doesn't influence the PPV level.

The wall pressure is related to the explosion pressure p_e by

$$p_h = p_e \cdot f^{2\gamma}. \quad (12)$$

Hustrulid et al. (1992, 1994) state that p_e and γ can be supplied by the explosives manufacturers. In one case (1992) they use $2\gamma = 3,33$ and in the other 2,6. In order to calculate p_e and γ one would have to know the ingredients of the explosive and to use codes like Tiger (Cowperthwaite and Zwisler, 1975) and Dyna (Hallquist, 1984). Some simplistic estimates can however be made.

According to the simplest detonation theory; one dimensional flow, a thin shock in which the chemical reaction is instantaneous and the discontinuity is steady (Fickett and Davis, 1979 or Persson et al, 1993), the CJ pressure of the detonation, p_{CJ} , is given by

$$p_{CJ}/g_o = \rho_e D^2/(\gamma+1). \quad (13)$$

From this point on $g_o = 1$ in SI units is omitted. Since $\gamma_{CJ} \approx 3$ for high densities the following approximation is often used

$$p_{CJ} = \rho_e D^2/4, \quad (14)$$

see Persson et al. (1993). At the CJ point, the explosive gases are compressed and, at a reexpansion to the original volume, the explosion pressure is thus

$$p_e = p_{CJ} \cdot [\gamma/(\gamma+1)]^\gamma \approx p_{CJ}/2, \quad (15)$$

since the postfactor lies within 0,42-0,45 when $1,5 < \gamma < 3$, see Persson et al. (1993) equation 4.34. Combining equations 13 and 15 we obtain

$$p_e = \gamma^\gamma/(\gamma+1)^{(\gamma+1)} \cdot \rho_e D^2 = n(\gamma) \cdot \rho_e D^2. \quad (16)$$

The prefactor in this equation lies in the interval 0,19-0,11 when $1,5 < \gamma < 3$. Here p_e becomes the bore-hole wall pressure p_h when the charge is fully coupled.

One possibility to estimate γ roughly is given by Fickett and Davis (1979) equation 2A-21 which says that

$$\gamma = \sqrt{(1+D^2/2e)}, \quad (17)$$

where $e(\text{J/kg})$ is the heat of complete reaction for a detonation in a polytropic gas. In turn, e may in an engineering sense be estimated by the available explosive energy given by the explosives manufacturer. Alternatively, γ as an influencing parameter in the dimensional analysis scheme in section 2.1 could be replaced by e .

There are several ways to take the effect of decoupled charges into account. Persson et al. (1993) use $2\gamma \approx 3$ in equation 12 to obtain

$$p_h = p_e \cdot f^3. \quad (18)$$

In the traditional design of presplit blasts, see Calder (1977) e.g. it is stated that the coupled bore-hole pressure is given by

$$p_h = n \cdot \rho_e D^2 \cdot f^{2.4}. \quad (19)$$

The prefactor n depends on explosive density and it is given in the form of a $n(\rho)$ diagram. A good curve fit in SI units is given by

$$n = 0,235(\rho/1000)^{-0,57} - 0,08 \text{ when } 0,8 \leq \rho \leq 2,0. \quad (20)$$

Then n lies in the interval 0,19-0,08. The value 2,4 of the f -exponent in equation 19 appears to go back to work in which decoupled spherical charges were fired in water filled metal drums and the pressure calculated from the shock velocity (Bauer, 1967). Note though that this experiment refers to a spherical decoupling and equation 19 to a cylindrical one and yet that the exponent is the same.

The connection between PPV_c and a critical stress level σ_c is tenuous at best. The one-dimensional relation between codirectional tensor and vector components, $\sigma = \rho c PPV$, can only be expected to hold at the front of a three dimensional P-wave which impinges on a material at rest. Furthermore this stress is compressive for an explosively generated wave. Behind the wave front, the radial expansion eventually sets up a tensile tangential stress field which can initiate radial cracks.

Thus, rather than trying to revise the PPV approaches to account for the observed cracking in the Vånga tests, the following simplistic approach was tried in Olsson and Bergqvist (1996a). It was assumed that

$$p_h = 0,125 p_e D^2 \cdot f^{2\gamma} \text{ and} \quad (21a)$$

$$R'_c = R_c - \varnothing_h/2 = k \cdot p_h^\beta. \quad (21b)$$

Then k and the exponents $2\gamma = \alpha$ and β were constants determined by a least squares fit to average crack length data for 7 different sets of experiments, see Table 2 or Olsson and Bergqvist (1996a), Table 21. The result of the fitting was that

$$\alpha = 3,37, \beta = 0,68 \text{ and } k = 1,52 \text{ when } p_h \text{ is given in MPa and } R'_c \text{ in cm.} \quad (22)$$

R'_c denotes that the crack length was measured from the bore-hole wall, not the center. A plot of the results is shown in Figure 3. The agreement between the line and the

Table 2: Data for crack length calculation with equations 21.

Explosive name	Charge diameter $\varnothing_c(\text{mm})$	Hole diameter $\varnothing_h(\text{mm})$	Charge concentr. $q(\text{kg/m})$	VOD $D(\text{m/s})$	Coupling ratio f	Crack length $R'_c(\text{cm})$
Gurit	17	51	0,21	2000	0,33	05
Gurit	17	38	0,21	2000	0,45	15
Gurit	22	64	0,40	2000	0,34	15
Gurit	22	24	0,40	2000	0,92	90
Emulet 20	51	51	0,45	1850	1,00	35
Kimulux 42	22	64	0,37	4800	0,34	25
Detonex 80	9	51	0,08	6500	0,21	15

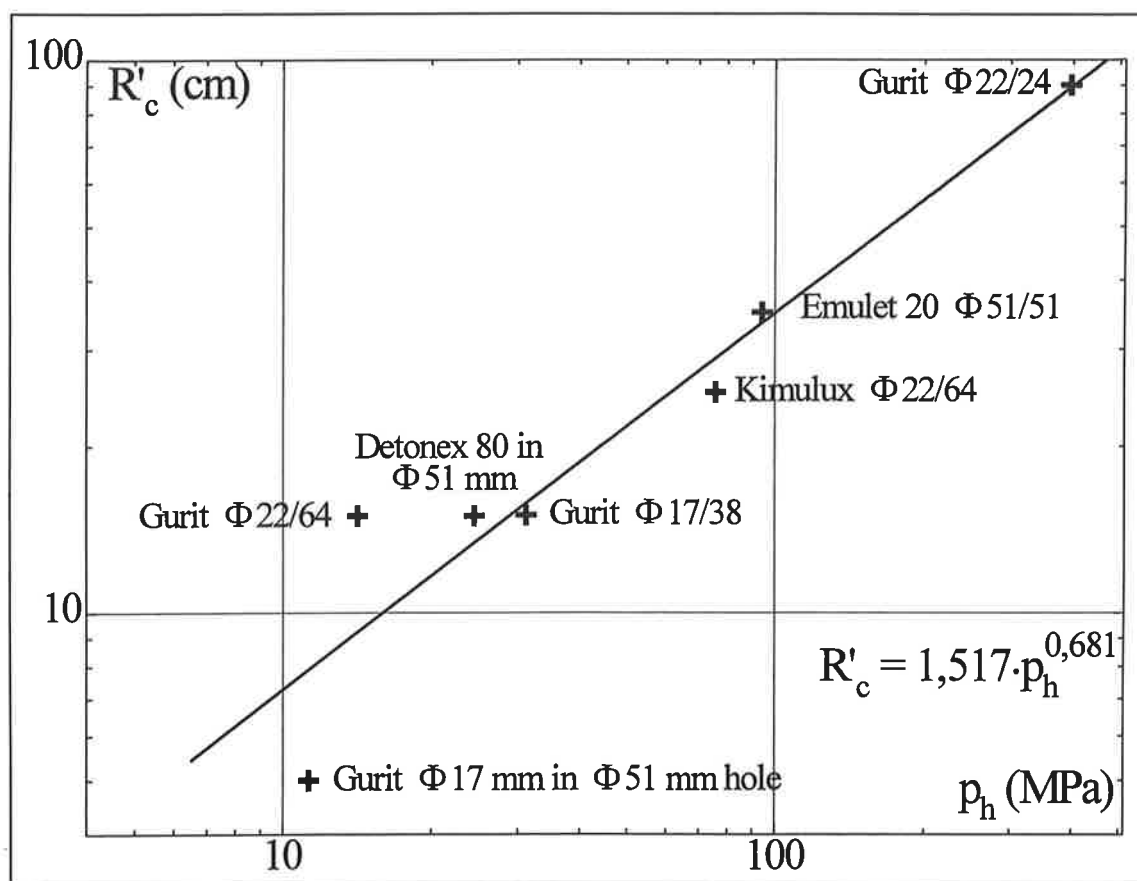


Figure 3: Curve fit diagram for results from Olsson and Bergqvist (1996a).

measured values is expressed by the coefficient of determination $r^2 = 0,899$. It could be better for short cracks, but the exponent 3,37 is not far from the value 3 in equation 18 and the exponent $0,68 < 1$ is consistent with Hustrulid's equation 11 when $l \neq 0$. This was considered to be encouraging and it became the starting point for a more systematic search for a crack length equation.

The dimensional form of equation 21b is awkward. By the simple introduction of parameters $\emptyset_h$ and K_{Ic} it can however be made dimensionless and thus serve as a first approximation of the function h in equation 3. In the sequel only non-dimensional crack length equations will be used.

2.3 Derivation of new crack length equation

Consider a cylindrical hole in an elastic medium subjected to a dynamic pressure pulse. According to Grady and Kipp (1987), the number of cracks generated along the circumference is given by

$$N = \pi \emptyset_h \cdot (\rho c \dot{\epsilon} / 6 K_{Ic})^{2/3}, \quad (23)$$

where $\dot{\epsilon}$ is the strain rate associated with the loading. The driving force for the fragmentation is the kinetic energy with respect to the center of mass and our choice is to represent it by the circumferential strain rate, i.e. $\dot{\epsilon} = \dot{\epsilon}_\theta$, rather than by the radial one. Further, according to Liu and Katsabanis (1993), a conservation of momentum across the shock wave requires that the pressure amplitude

$$p_s = \rho \dot{u}_r \dot{u}_s, \quad (24)$$

where $\dot{u}_r$ is the radial particle velocity and $\dot{u}_s$ the shock velocity. The latter two are related through the shock Hugoniot,

$$\dot{u}_s = c + s \cdot \dot{u}_r, \quad (25)$$

where c is the elastic P-wave velocity in rock, see above. For rocks $c \approx 3000$ - 6000 m/s and the slope $s \approx 1$ - 2 . For reasonably low loadings, such as in cautious blasting, $\dot{u}_s \approx c$ and thus the radial particle velocity becomes

$$\dot{u}_r = p_s / \rho c. \quad (26)$$

By definition the circumferential strain $\varepsilon_\theta = u/r$ where r is the radial coordinate. Hence

$$\dot{\varepsilon}_\theta = \dot{u}/r. \quad (27)$$

Combining equations 26 and 27 one obtains

$$\rho c \dot{\varepsilon}_\theta = p_s/r \text{ or} \quad (28a)$$

$$\rho c \dot{\varepsilon}_\theta = 2p_s/\varnothing_h \quad (28b)$$

at the bore-hole wall.

Inserting the latter expression, equation 28b, into equation 23 yields

$$N = \pi/3^{2/3} \cdot (p_h \sqrt{\varnothing_h/K_{Ic}})^{2/3} \quad (29)$$

and by requiring that $N \geq 2$ we have obtained an expression for the minimum dynamic pressure required to split the bore-hole by initiation of cracks

$$p_h \geq p_{h,crit} = 1,52 \cdot K_{Ic} / \sqrt{\varnothing_h}. \quad (30)$$

For a $\varnothing_h = 50$ mm bore-hole in granite with $K_{Ic} \approx 2$ MPa $\sqrt{m}$ (Ouchterlony, 1991) we obtain $p_{h,crit} \approx 14$ MPa which is quite reasonable. A combination of equations 29 and 30 yields

$$N = 2(p_h/p_{h,crit})^{2/3}. \quad (31)$$

To initiate 4 cracks dynamically would thus require $2\sqrt{2} \cdot p_{h,crit}$ or about 38 MPa.

Note the difference between equation 30 and the corresponding static fracture mechanics expression,

$$p_h \geq p_{h,crit} = K_{Ic}/2,24\sqrt{(\pi a_c)}. \quad (32)$$

This expression is based on the pressurization of flaws of a critical depth a_c on the bore-hole wall (Ouchterlony, 1983) and it is independent of hole size.

In order to asses the crack arrest situation, consider a static star crack with N arms of length R_c , subjected to pressure at the origin only. A vast majority of the remaining radial

cracks in the Vånga tests are not open to gas penetration, see Figure 1 above, so this assumption is reasonable. According to Ouchterlony (1983)

$$K_I \approx \pi/(2\sqrt{N}) \cdot p \cdot \sqrt{\pi R_c}. \quad (33)$$

The cracks will arrest, i.e. no longer grow when $K_I \leq K_{Ia} \approx K_{Ic}$. For a given bore-hole pressure p_h , the maximum crack length would thus be

$$R_c/\sqrt{\pi} \leq \pi/4N \cdot (p_h \sqrt{\pi}/K_{Ic})^2. \quad (34)$$

The exponent 2 for p_h in this expression is however far from the value $0,68 \approx 2/3$ obtained in equation 21 for the Vånga results. In order to obtain the desired exponent value, one would have to have $K_I \propto R_c^{-3/2}$ such as for the penny shaped crack with concentrated forces at the origin.

Since this geometry is quite unrealistic, it was decided to extend the arguments leading to equations 29 and 30 instead. To do this we need an equation for the shock wave amplitude travelling outward from a cylindrical blast-hole. Liu and Katsabanis (1993) provide this. With some simplifications, their main results may be written

$$p_h \approx 0,167 p_s D^2 \cdot (p_c/p_s D)^{0,25} \text{ and} \quad (35a)$$

$$p_s/p_h = (\sqrt{\pi} r/2)^{\delta} \text{ where } \delta \approx 1,5(D/c)^{0,25}. \quad (35b)$$

Here p_s is the shock amplitude at r . Actually, Liu and Katsabanis (1993) give a prefactor of 1,5 in equation 35a instead of $0,167 \approx 1/6$, but that is unnaturally high, c.f. equations 16 and 20. A perusal of their data shows that a much better fit is obtained by the prefactor $1/6$.

Equation 35a introduces the concept that an "impedance ratio" between explosive and rock influences the result. Assuming that there exists a critical shock amplitude $p_{s,crit}$, an inversion of equation 35b yields at $r = R_c$, the critical distance

$$R_c/\sqrt{\pi} = 0,5 \cdot (p_h/p_{s,crit})^{1/\delta} \text{ where } 1/\delta \approx 2/3 \cdot (c/D)^{0,25}. \quad (36)$$

For many explosives $D \approx c \approx 5500$ m/s, so a normal value for the exponent is $2/3$ like in equation 22. Further, the exponent 0,25 makes the c/D factor relatively insensitive. For a low VOD explosive with $D = 2000$ m/s it would become $2/3 \cdot (5,5/2)^{0,25} = 0,86$ which is not too far from $2/3$ e.g. Thus Liu and Katsabanis' (1993) equations are quite attractive.

If we interpret equation 23 at $2R_c$ instead of at $\varnothing_h$, use that $N \geq 2$, and equations 28 and 30, we may write the critical dynamic pressure $p_{s,crit}$ at R_c as

$$p_s \geq p_{s,crit} = 2p_{crit} \cdot \sqrt[3]{(\varnothing_h/2R_c)} = 1,08 \cdot K_{lc} \cdot \sqrt[3]{R_c}. \quad (37)$$

Insertion into equation 35b gives

$$p_h \geq p_{h,crit} = 1,08 \cdot K_{lc} \cdot \sqrt[3]{R_c} \cdot (2R_c/\varnothing_h)^5. \quad (38)$$

Using the results in equations 17, 35 and 38 we can extend the simplistic approach of equations 21a-b to the following set of equations

$$p_h = n(\gamma) \rho_e D^2 \cdot (\rho c / \rho_e D)^{0,25} \cdot f^\alpha \text{ and} \quad (39a)$$

$$(2R_c/\varnothing_h)^{(D/c) \uparrow 0,25} = \lambda (p_h \sqrt[3]{R_c} / K_{lc})^\beta. \quad (39b)$$

Here α , β and λ are constants to be determined e.g by a logarithmic least squares fit to the crack length data from Vånga. Possibly we should choose $\alpha = \alpha_0 \gamma$, c.f. equations 11 and 21. Equations 39 represent a physically based estimate of the functional form of h in equation 3. Unfortunately $R_c/\varnothing_h$ isn't given directly in closed form but once α and β have been determined, the equations can be rearranged to do so, c.f. section 3.2.

A comparison of equations 39 and 3 shows that the parameter most clearly representing the charge concentration q , i.e. $\rho_e \varnothing_e^2 / \rho \varnothing_h^2$, is missing. It can however be reconstructed from the impedance ratio $\rho c / \rho_e D$, the coupling ratio f and the velocity ratio c/D . Further, the parameter $B/\varnothing_h$ is also missing but the reason for that is in part that the Vånga results seem to be rather insensitive to this parameter, see section 3.1 below.

3 NUMERICAL RESULTS

3.1 Curve fitting procedure

Equations 39 are relatively complicated compared to equations 21 or any of the other simple formulas in section 2.1. An important question is now whether the experimental data can support their degree of complexity or not regarding $n(\gamma)$, the impedance factor $(\rho c / \rho_e D)^{0,25}$, the exponent $\alpha = \alpha_0 \gamma$, the exponent $(c/D)^{0,25}$ and the change from $\varnothing_h$ to R_c as the length scale in the dimensionless pressure parameter in (39b).

To answer this question we first write equations 21 in dimensionless form

$$p_h = 0,167 \rho_e D^2 f^\alpha \text{ and} \quad (40a)$$

$$2R_c/\phi_h = \lambda [p_h \sqrt{(\phi_h/2)/K_{lc}}]^\beta \quad (40b)$$

to use as a reference. The crack length data obtained at Vånga are given by Table 2, see Olsson and Bergqvist (1996a). They represent, in slightly rounded numbers, the average crack lengths measured from the half-casts of the middle one or two blast-holes of groups of holes which have been fired simultaneously. This reduces possible edge effects and ensuing scatter in the data. Since each group consists of 3-4 holes this also reduces the number of available crack length data.

The data in Table 2 was obtained using different combinations of B·S; 0,5·0,5 m, 0,8·0,5 m, 0,8·0,8 m and 1,0·0,8 m, so neither S/B nor B/φ_h has been constant, c.f. equation 3. A large majority of the data represents the case 0,5·0,5 m however and the influence of S/B and B/φ_h doesn't appear to be large. Thus, their effects are left out until the data base is sufficiently large for them to be more rigorously judged. The number of data behind Table 2 is nevertheless well over 50 if all combinations of B·S are used.

Table 3: Data for explosives tested at Vånga

Explosive name	VOD D(m/s)	Energy cont. e(MJ/kg)	Density ρ _e (kg/m ³)	Adiab.exp. γ	Prefactor n(γ)
Gurit	2000	3,50	1000	1,254	0,213
Kimulux 42	4800	3,20	1100	2,145	0,140
Emulet 20	1850	2,35	250	1,315	0,205
Detonex	6500	5,95	1250	2,133	0,141

The necessary explosives data is given in Table 3. The material parameters chosen for the Vånga granite are

$$\rho = 2650 \text{ kg/m}^3, c = 5500 \text{ m/s and } K_{lc} = 2 \text{ MPa}\sqrt{\text{m}}. \quad (40c)$$

Since the tests have only been made in this one rock, the correct values become important first when crack lengths in different rocks are compared.

The curve fitting algorithms have been implemented in Mathcad Plus 6.0 (Deng, 1996). The result of the curve fit is that in equations 40,

$$\alpha = 3,74, \beta = 0,72 \text{ and } \lambda = 4,18. \quad (40d)$$

These numbers differ a little from those in equation 22, both because they now refer to a crack length which is measured from the center of the bore-hole and because equation 40b is dimensionless. The goodness of fit, as measured by the coefficient of determination r^2 is 0,857 which will be our reference. See Figure 4.

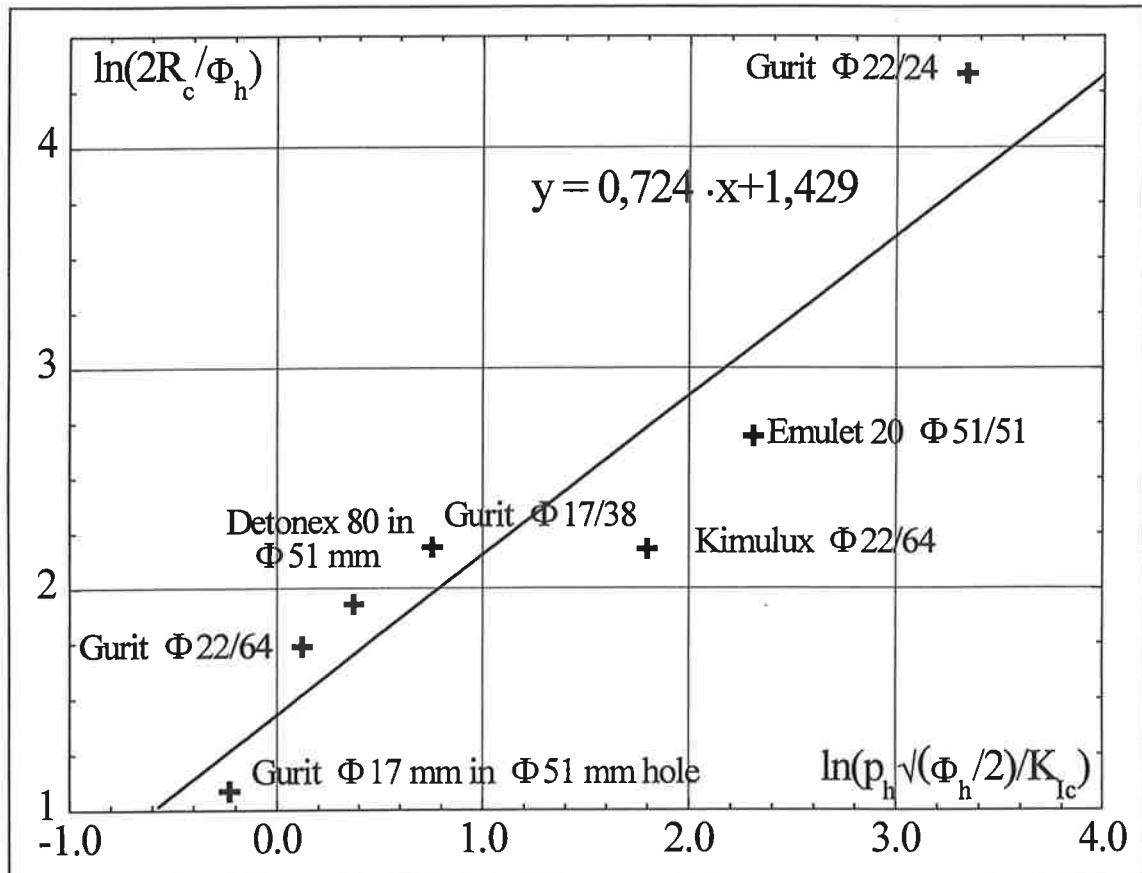


Figure 4: Equation 40b with optimal parameters fitted to the Vånga data.

First we change from Φ_h to R_c as the length scale in the dimensionless pressure parameter in equation 40b to obtain

$$p_h = 0,167 \rho_s D^2 f^\alpha \text{ and} \quad (41a)$$

$$2R_c / \Phi_h = \lambda (p_h \sqrt{R_c} / K_{lc})^\beta. \quad (41b)$$

The result of the curve fit is that

$$\alpha = 3,54, \beta = 0,58, \lambda = 3,32 \text{ and } r^2 = 0,926. \quad (41c)$$

The improved curve fit thus supports the change of pressure length scale which the new crack length equation has introduced.

For the remaining curve fitting work it is convenient to write the crack length equations 39 as follows

$$p_h = n(\gamma) \rho_e D^2 i(\rho c / \rho_e D) f^{\alpha a(\gamma)} \text{ and} \quad (42a)$$

$$(2R_c / \phi_h)^{e(D/c)} = \lambda (p_h \sqrt{R_c / K_{Ic}})^{\beta}, \quad (42b)$$

where the functions a, e, i and n are either constants or the functions introduced in section 2.3, see equations 39 e.g. Starting with the combination of functions represented by equation 41, case a below, the combinations a-i in Table 4 have been tried.

Table 4: Combinations of functions used in curve fitting procedure

Parameter combination case no	Detonation prefactor n	Impedance factor i	Decoupling exponent a	Crack length exponent e
a	1	1	1	1
b	$\gamma^2 / (\gamma + 1) \gamma^{\gamma+1}$	1	1	1
c	1	$(\rho c / \rho_e D)^{0,25}$	1	1
d	1	1	1	$(D/c)^{0,25}$
e	$\gamma^2 / (\gamma + 1) \gamma^{\gamma+1}$	1	γ	1
f	1	$(\rho c / \rho_e D)^{0,25}$	1	$(D/c)^{0,25}$
g	$\gamma^2 / (\gamma + 1) \gamma^{\gamma+1}$	1	1	$(D/c)^{0,25}$
h	$\gamma^2 / (\gamma + 1) \gamma^{\gamma+1}$	$(\rho c / \rho_e D)^{0,25}$	1	$(D/c)^{0,25}$
i	$\gamma^2 / (\gamma + 1) \gamma^{\gamma+1}$	$(\rho c / \rho_e D)^{0,25}$	γ	$(D/c)^{0,25}$

The results are shown in Table 5. The coefficient of determination increases relatively much as the degree of complexity of the curve fitting equations increases. Since the functions n , i , a and e were not arbitrary but motivated by a model, the outcome of the curve fitting supports the model. It can not be ruled out though that another choice of adding complexity, i.e. functions, would result in a still better fit, but to look for that is premature before the whole Vånga data set has been used.

Table 5: Resulting parameter values form curve fitting procedure

Parameter combination case no	Decoupling exponent α	Pressure exponent β	Pressure prefactor λ	Coefficient of determination r^2
ref	3,74	0,72	4,18	0,857
a	3,54	0,58	2,32	0,926
b	3,02	0,67	1,26	0,953
c	3,02	0,64	1,09	0,915
d	2,55	0,59	0,97	0,957
e	1,16	1,01	0,19	0,887
f	2,12	0,66	0,44	0,953
g	2,20	0,67	0,566	0,969
h	1,76	0,77	0,198	0,975
i	0,73	1,03	0,038	0,915

The largest increase in r^2 has been obtained by changing the length scale in the dimensionless pressure parameter; ref case $\rightarrow$ case a. The second largest increase was obtained by then adding either a variable detonation prefactor $n(\gamma)$, case a $\rightarrow$ case b, or a variable crack length exponent $e(D/c)$, case a $\rightarrow$ case d.

Adding the bore-hole impedance factor $i(p_c/p_o D)$ initially has a slightly negative influence on the curve fit since r^2 decreases somewhat; case a $\rightarrow$ case c and case d $\rightarrow$ case f. In the end the effect is slightly positive; case g $\rightarrow$ case h. Using a variable decoupling exponent $a(\gamma)$ finally reduces the quality of the curve fit considerably; case b $\rightarrow$ case e and case h $\rightarrow$ case i.

At this stage, the decoupling exponent should thus be kept constant in the final crack length prediction equation, i.e. we set $a = 1$. Table 4 also shows that the pressure exponent β is relatively constant, $\beta = 0,67 \pm 15\%$, if cases e and i are excluded. This β -value thus has a relatively high credibility.

Next, consider whether the impedance factor i should be included or not. Both i and the crack length exponent e stem from the work of Liu and Katsabanis (1993) so it could be argued that both or neither of them should be included.

There are several reasons why i should be excluded at this stage, i.e. set to $i = 1$. Firstly it has a marginal effect on the curve fit; compare case g with $r^2 = 0,969$ with case h with $r^2 = 0,975$. Secondly the number of data points on which our results are based is small. Thirdly, the decoupling exponent $\alpha = 2,2$ for case g comes very close to the value 2,4 frequently used in the literature (Calder, 1977).

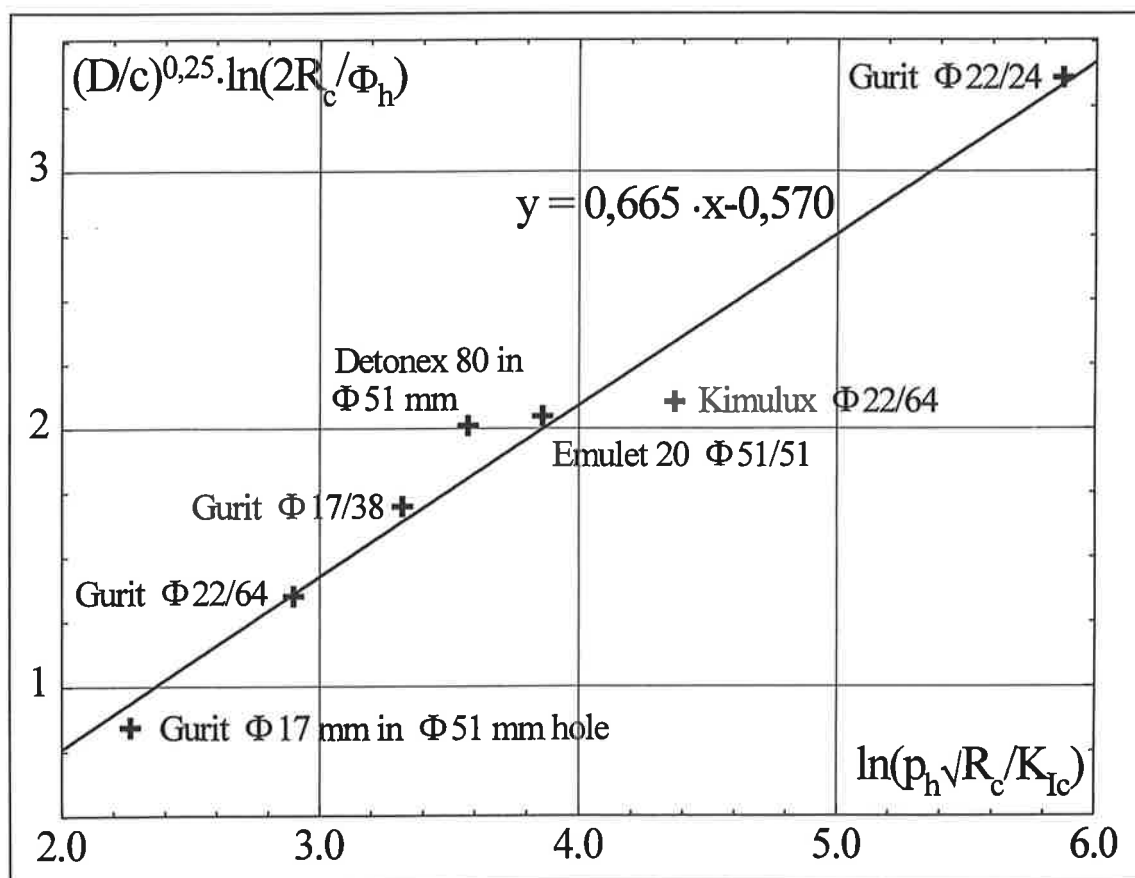


Figure 5: "Best" crack length prediction equation, case g, and Vånga data.

The conclusion is thus that presently the "best" crack length prediction equation is given by **case g** and

$$p_h = \gamma/(\gamma+1)^{(\gamma+1)} \cdot p_e D^2 \cdot f^{2,2} \text{ and} \quad (43a)$$

$$(2R_c/\varnothing_h)^{(D/c)\uparrow 0,25} = 0,566 \cdot (p_h \sqrt{R_c/K_{lc}})^{0,67}. \quad (43b)$$

A comparison of equation 43b with the data is shown in a log-log diagram in Figure 5 above. The vertical deviation of any average is about $\pm 20\%$. The Kimulux average has a higher pressure (or shorter crack length) than the fitted line, the Detonex average has a lower pressure (or longer crack length). The individual values naturally deviate more and the standard deviations for the measured data are discussed in section 4.1.

The exclusion of the impedance factor i in equation 42a, would seem to mean excluding q from our influence parameters. This conclusion is somewhat misleading however since an increase in charge size for a given explosive and at constant f means an increase in bore-hole size $\varnothing_h$ and thus in crack length R_c , see also section 4.2.

3.2 Approximate design diagram

Equations 43 form an implicit relationship between crack length R_c and the other blasting parameters which is easily rearranged to an explicit one. If we let $0,67 \approx 2/3$, then

$$p_h = \gamma/(\gamma+1)^{(\gamma+1)} \cdot p_e D^2 \cdot f^{2,2} \text{ and} \quad (43a)$$

$$2R_c/\varnothing_h = [0,428 \cdot p_h \sqrt{(\varnothing_h/2)/K_{lc}}]^{2/[3(D/c)\uparrow 0,25-1]}. \quad (43b')$$

The validity of equation 43b' is restricted to $R_c \geq \varnothing_h/2$, because cracks inside the bore-hole do not exist. For cracking to occur at all, the bore-hole pressure must be larger than some critical value, $p_h \geq p_{h,crack}$. Thus 43b' may be rewritten

$$2R_c/\varnothing_h = (p_h/p_{h,crack})^{2/[3(D/c)\uparrow 0,25-1]}, \text{ where} \quad (43b'')$$

$$p_{h,crack} = 3,30 \cdot K_{lc} / \sqrt{\varnothing_h}. \quad (43c)$$

This cracking pressure is about twice as high as the theoretically predicted value $p_{h,crit}$ from equation 30 which has the prefactor 1,52. Thus theory and experiments are not quite consistent in this respect.

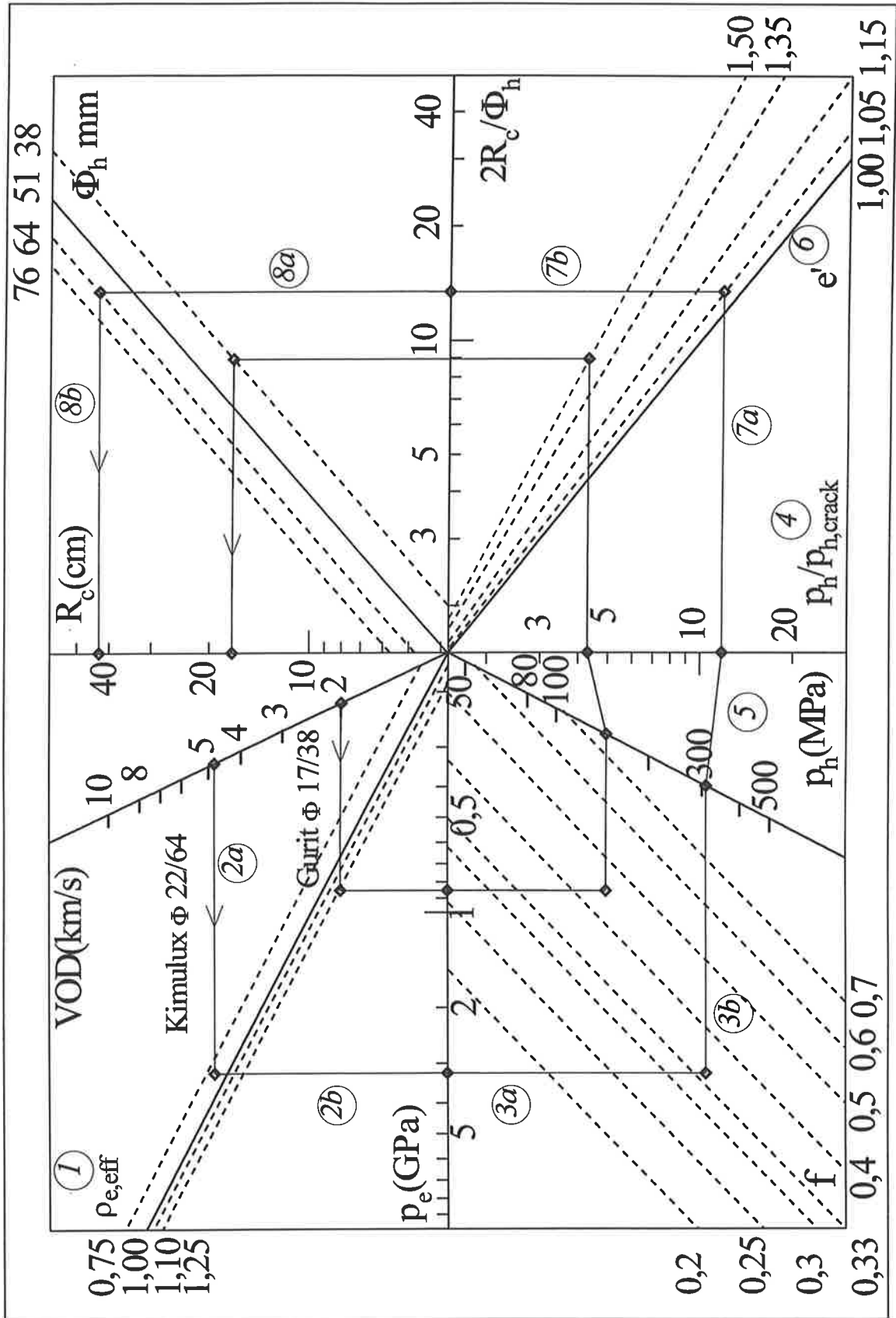


Figure 6: Approximate design diagram for cautious blasting, based on equation 43.

Equations 43 are easily implemented on a computer or a hand-held calculator. There is however also some merit in constructing a design diagram such as in Figure 6 above in which the axes are logarithmic. It gives an immediate feeling for the relative importance of the different parameters, see below.

The different steps in the going through the design diagram are as follows, see the encircled numbers in Figure 6:

1. Calculate firstly γ from the explosive properties and equation 17 and secondly an effective density for the explosive,

$$\gamma = \sqrt[3]{(1+D^2/e)} \text{ and} \quad (17)$$

$$\rho_{e,eff} = 6\gamma^2/(\gamma+1)]^{(\gamma+1)} \cdot \rho_e = 6n(\gamma)\rho_e, \quad (44)$$

see Table 3 e.g. for explosives tested at VÅnga.

2. Start on the VOD-axis at the given D-value and draw a straight line leftward to the appropriate density line, $\rho_{e,eff} = 0,75, 1,0, 1,10$ and $1,25$. Continue vertically downward to the negative horizontal axis and read off the explosion pressure p_e .
3. Continue vertically downward to the appropriate decoupling line, $f = \text{constant}$, and then draw a horizontal line rightward to the negative slanting pressure axis. Read off the bore-hole pressure p_h .
4. Calculate the critical bore-hole pressure for cracking from equation 43c

$$p_{h,crack} = 3,30 \cdot K_{lc} / \sqrt{\phi_h}, \quad (43c)$$

and divide p_h by this number to obtain the non-dimensional overpressure number $p_h/p_{h,crack}$.

5. Move from the negative pressure axis to the appropriate value of the overpressure number on the negative vertical axis.
6. Calculate the pressure exponent

$$e' = 2/[3(D/c)^{0,25}-1] \quad (45)$$

7. Draw a straight line horizontally from the overpressure axis rightward to the appropriate relative damage line given by e' . Then draw a vertical line upward to the positive horizontal axis and read off the relative crack length $2R_c/\varnothing_h$.
8. Continue vertically upward to the appropriate bore-hole diameter line. Then draw a horizontal line leftward to the vertical damage axis and read off the depth of the damage zone, R_c , in cm.

In Figure 6, two design examples are given, a $\varnothing_e = 22$ mm charge of Kimulux 42 in a $\varnothing_h = 64$ mm hole and a $\varnothing_e = 17$ mm charge of Gurit in a $\varnothing_h = 38$ mm hole. In this figure, the influence of D is given both by the starting point on the VOD-axis and the slope e' of the damage lines in the 3rd quadrant. For a given explosive, the value of $p_{e,eff}$ may be calculated once and for all. For the ones used here the value may be obtained directly from equation 44 and Table 2.

The substantial influence of the decoupling is given by the lines in the 2nd quadrant. The influence of absolute hole size doesn't enter until the calculation of the critical bore-hole pressure $p_{h,crack}$ in equation 43c. In Figure 6 the move from the bore-hole pressure axis to the overpressure axis will be horizontal for a $\varnothing_h = 51$ mm hole in rock with a fracture toughness of $K_{Ic} = 2$ MPa $\sqrt{m}$.

In the 3rd quadrant, the influence of the VOD is felt again through the pressure exponent e' from equation 45. It is visible in the slope of the prediction line and can for given explosive-rock combination be calculated before hand. The influence of bore-hole size enters both into the value of $p_{h,crack}$, which decreases with size, and more directly into the lines in the 4th quadrant.

4 DISCUSSION AND CONCLUSIONS

4.1 Discussion of results

A pure dependence on charge concentration q is not to be generally expected since the combination $p_e \varnothing_e^2$ alone doesn't describe the explosive in equations 43 in this report. Firstly in equation 45, when $D = c$ then $e' = 1$ and when $D \approx 2000$ m/s and $c = 5500$ m/s then $e' = 1,5$. Next, using equation 17 to replace D^2 in equation 43a by the explosive energy e , there results for the following two cases

$$R_c \approx \text{const} \cdot q \cdot e / \varnothing_e^{0,5} \cdot f^{0,7} \text{ for } D = c \text{ and} \quad (46a)$$

$$R_c \approx \text{const} \cdot (q \cdot e)^{1,5} / \varnothing_e^{1,25} \cdot f^{1,55} \text{ for } D = 0,36 \cdot c. \quad (46b)$$

For a given explosive however for which $p_e = \text{constant}$, $q \propto \varnothing_e^2$ and the net effect of equations 46 is that $R_c \approx \text{const} \cdot q^{0,75-0,88}$ which is slightly more forgiving for large charges than the linear relationship postulated by equation 6. On the other hand a slow burning explosive would seem to be relatively more sensitive to decoupling than a fast burning one. This may seem counterintuitive but it is a result of Liu and Katsabanis (1993) pressure decay relation, equation 35b. The constant depends on γ so a comparison between the two equations for different explosives is not immediately apparent.

As mentioned in section 3.1, two parameters in the crack length prediction equations have a certain degree of arbitrariness, c and K_{Ic} . Unless experiments are made in another type of rock, one can not be sure that these are the material parameters that will describe our test results best and that correct values for them have been chosen.

Because of equations 30 and 43c, K_{Ic} is the easiest to treat. Instrumented dynamic pressure strength tests for bore-holes would validate or falsify the size effect implied by these equations. Further, another value for K_{Ic} than 2 MPa $\sqrt{m}$ would change the value of the prefactor λ in the right member in equation 43b but leave the prediction itself undisturbed. For fracture toughness measurements there are international standard testing methods valid for isotropic rock (ISRM, 1988 and 1995).

The matter of c is different because of the form chosen for the exponent in the left member of equation 43b. Another value of c than 5500 m/s would probably influence all the curve fit constants α , β and λ and consequently the crack length prediction.

A comparison of the measured crack lengths and the values predicted by the design diagram or the formulas in equations 43 are given in Table 6. An effective charge diameter for Detonex 80 of $\varnothing_e \approx 9$ mm has been used, giving a coupling ratio of $f = 0,18$. The corresponding data and crack length intervals are also shown in Figure 7. Note that two of the points are represented by single data; $\varnothing_e = 17$ mm Gurit in $\varnothing_h = 38$ mm holes and $\varnothing_e = 22$ mm Gurit in $\varnothing_h = 24$ mm holes. Their positions could thus contain relatively substantial errors. Note also however that the prediction line passes more or less straight through all the points representing the low VOD explosives, Gurit and Emulet 20.

In Table 6, three of the seven points deviate more than 20% from the prediction value; $\varnothing_e = 17$ mm Gurit in $\varnothing_h = 51$ mm holes, $\varnothing_e = 22$ mm Kimulux 42 in $\varnothing_h = 64$ mm holes, and Detonex 80g/m in $\varnothing_h = 51$ mm holes. In practice though, a 3,1 cm overestimate of the average crack lengths behind $\varnothing 17$ mm Gurit in $\varnothing 51$ mm holes is insignificant.

Table 6: Comparison of measured and calculated crack lengths

Explosive name	Hole diameter $\Phi_h(\text{mm})$	Coupling ratio f	Measured cracks $R_{c,m}(\text{cm})$	Predicted cracks $R_c(\text{cm})$	Difference %
Gurit	51	0,33	7,6	10,7	+41%
Gurit	38	0,45	16,9	16,9	$\pm 0\%$
Gurit	64	0,34	18,2	17,6	-3%
Gurit	24	0,92	91,2	81,3	-11%
Emulet 20	51	1,00	37,6	41,6	+11%
Kimulux 42	64	0,34	28,2	47,5	+69%
Detonex 80	51	0,18	17,6	13,4	-24%

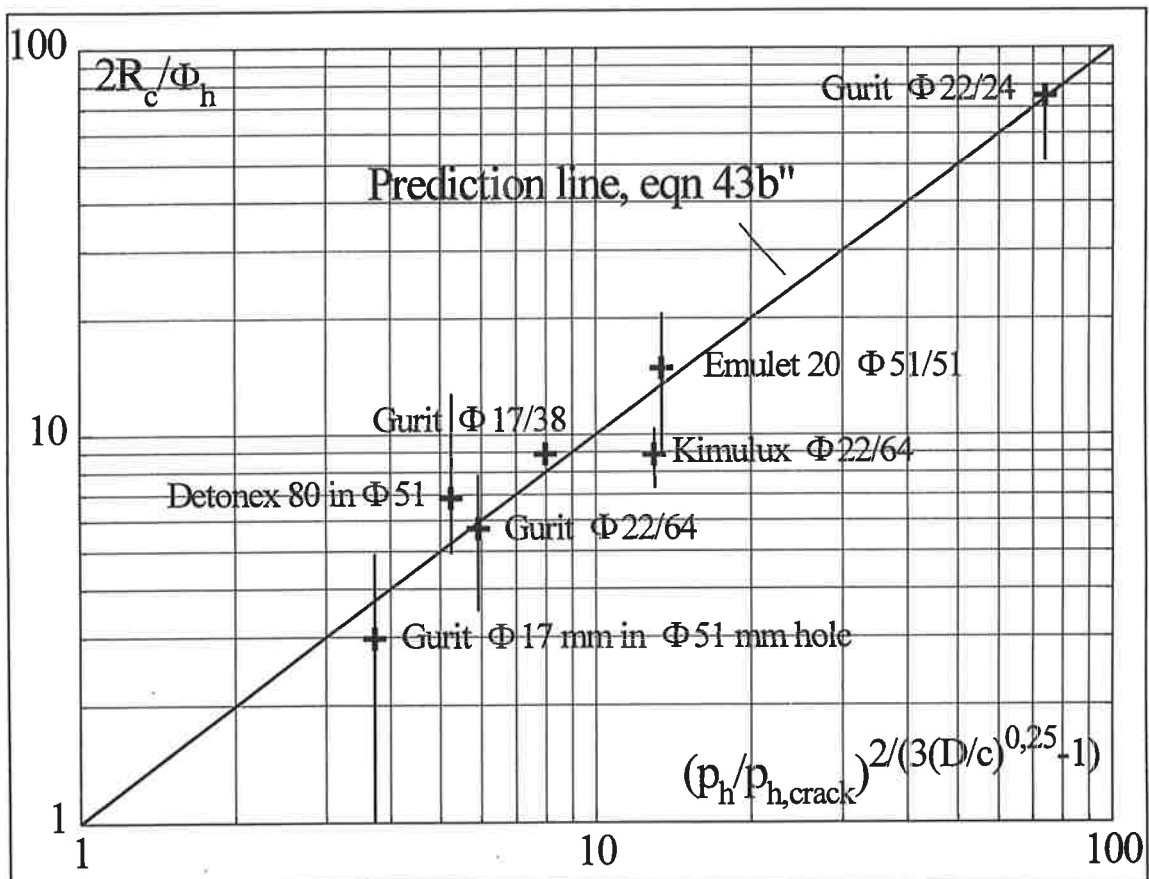


Figure 7: Crack length prediction line, equations 43, and ranges of Vângà results.

The two high VOD explosives Kimulux and Detonex are the outliers. Kimulux is the more severe case because the prediction line in Figure 7 above overestimates the measured average crack lengths by 69% and the whole crack length interval, mean $\pm$ std deviation lies below the prediction line. On the other hand, a nonconservative estimate of the average crack lengths behind Detonex is not good either.

The Detonex and Kimulux points lie on each side of the prediction line so perhaps no common correction based on the D/c ratio alone can bring both back to the line? This may be the explanation why the attempt at using the decoupling exponent $\alpha = \alpha_o \gamma$ above failed to improve the curve fit and why the impedance factor $(\rho c / \rho_o D)^{0,25}$ was only marginally effective in the curve fitting procedure in section 3.1.

The explosive behavior of a straight emulsion like Kimulux 42 is relatively well known, that of the PETN cord with wrapping seemingly less so. Sanchidrián et al. (1996) use a "mock explosive" concept to describe the expansion properties of PETN cord, 3g/m, 6g/m and 12 g/m in $\varnothing = 32$ mm bore-holes. As a first step they calculate bore-hole wall pressures which become 58,5 MPa, 29,5 MPa and 14,5 MPa respectively. With assumed properties of their PETN according to our Table 2, our equation 43a gives the numbers 57,0 MPa, 27,2 MPa and 12,4 MPa which are relatively close. For 80 g/m PETN cord in $\varnothing_h = 51$ mm holes, Sanchidrián's method gives a bore-hole pressure of 154 MPa, equation 43b gives 163 MPa which is again close. Compared to this there seems to be no large error in our description, but the matter is raised again below.

Jokinen and Ylätaalo (1995 and 1996) report one way blasting tests in Oulainen gabbro and Ristijärvi granite in Finnish dimensional stone quarries. Rows of $\varnothing_h = 32$ mm holes behind a free surface were initiated simultaneously with 10 g/m detonating cord. The toe and the sides were free so the situation was very much like that at Vånga. They measured the crack lengths obtained with the same sawing and penetrant techniques that were used at Vånga. The difference is that they documented the cracks in the burden, not the ones behind the holes. The values given here are the longest radial cracks as measured from the bore hole wall and denoted L_{max} by Jokinen and Ylätaalo.

Data for their explosives are given in Table 7. K-17, F-17 and Formex are comparable to our low VOD explosives. Two or three pipe charges or cartridges tied to a 10 g/m cord downline were used per hole, giving a charge length of 0,8-1,4 m and 1,6-1,2 m uncharged without stemming, Jokinen (1997). No primer was used. Their PETN cord differs mainly in charge concentration. The density of the core has been assumed to be $\rho_o = 1250 \text{ kg/m}^3$, just like in Table 3 and the core diameter $\varnothing_o$ calculated accordingly. The cord downline has not been included in the K-17, F-17 and Formex numbers.

Table 7: Data for explosives used by Jokinen and Ylätaalo (1995, 1996). Bore hole diameter $\varnothing_h = 32$ mm.

Explosive name	VOD D(m/s)	Energy content e(MJ/kg)	Density $\rho_e(\text{kg/m}^3)$	Charge diameter $\varnothing_e(\text{mm})$	Coupling ratio f
K-17	1900	1,4	950	17	0,53
F-17	2500	2,8	950	17	0,53
Formex	1800	4,3	600	22	0,69
PETN 40 g/m	6500	5,95	1250	6,4	0,20
PETN 20 g/m	6500	5,95	1250	4,5	0,14

The maximum crack lengths obtained are shown in Table 8 and compared with the predictions that equations 43 give. Note that the values of Jokinen and Ylätaalo have been increased by the hole radius 1,6 cm to make them comparable. The rock data used was obtained from their report and by personal communication, Jokinen (1997).

- Oulainen gabbro: $\rho = 2900 \text{ kg/m}^3$, $c = 6900 \text{ m/s}$ and $K_{Ic} = 2,4 \text{ MPa}\sqrt{\text{m}}$,
- Ristijärvi granite: $\rho = 2660 \text{ kg/m}^3$, $c \approx 6100 \text{ m/s}$ and $K_{Ic} = 2,0 \text{ MPa}\sqrt{\text{m}}$.

Table 8: Comparison of crack lengths measured by Jokinen and Ylätaalo (1995, 1996) with the predictions of equations 43.

Explosive name	Oulainen gabbro			Ristijärvi granite		
	Measured cracks $R_{c,m}(\text{cm})$	Predicted cracks $R_c(\text{cm})$	Diff. %	Measured cracks $R_{c,m}(\text{cm})$	Predicted cracks $R_c(\text{cm})$	Diff. %
K-17	13,6	13,9	+2%	15,6	16,6	+6%
F-17	27,6	25,9	-6%	2,1	29,1	-
Formex	4,6	20,5	-	15,6	24,0	+54%
PETN 40 g/m	10,6	8,0	-25%	19,6	8,9	-55%
PETN 20 g/m	-	3,7	-	7,6	4,2	-45%

Equations 43 predict that the crack lengths in Ristijärvi granite should be slightly longer than in Oulainen gabbro. This agrees with the experiments except for the F-17 explosive in the granite, a point which Jokinen and Ylätaalo (Part 2, 1996) discard in their analysis. The agreement between measured and predicted crack lengths is not bad for the K-17 and F-17 explosives, considering that the downline hasn't been included. The measured Formex value in Oulainen gabbro seems to be too low, but it is based on only one hole.

Interestingly all three predicted crack lengths from the PETN cord holes in Table 8 are lower than the measured ones, just like our own results for the cord with a higher charge concentration, see Table 6. This strengthens the suspicion that our explosives description might be overly simple.

Recently, Brent (1995) has circumvented all the simplifications in the explosives description in this report by using proprietary fundamental detonation codes which can model the non-ideal detonations in commercial explosives under the actual confinement and diameter. This amounts to saying that the value of γ depends on the pressure level, or density, in a way which is unique for each explosive. For ANFO he gives a range of $2,8 \rightarrow 1,3$ as the pressure approaches atmospheric conditions, for a doped emulsion $>3,0 \rightarrow 1,3$ and presumably a higher decoupling exponent. Our decoupling exponent 2,2 corresponds to a γ -value of 1,1 which implies that equation 43a might underestimate the effect of decoupling on the bore-hole pressure considerably.

Even if Brent doesn't give the explosive energy data for his two examples with decoupled charges a comparison can be attempted. His first example is the use of an ANFO-based explosive with $\rho_e = 800 \text{ kg/m}^3$ and a measured VOD of $D = 4360 \text{ m/s}$ in bore-holes with $\varnothing_h = 270 \text{ mm}$. The designed bore-hole pressure is $p_h = 40 \text{ MPa}$ which requires a coupling factor of $f = \sqrt{0,12} \approx 0,35$. If we estimate the explosive energy as 3,5-4,0 MJ/kg (Persson et al., 1993) then equation 43a yields $p_h = 230\text{-}240 \text{ MPa}$.

Brent's second example (1995) is the use of what could be an emulsion explosive with $\rho_e = 1230 \text{ kg/m}^3$ and a measured VOD of $D = 4790 \text{ m/s}$ in bore-holes with $\varnothing_h = 89 \text{ mm}$. The designed bore-hole pressure is $p_h = 112 \text{ MPa}$ which this time requires a coupling factor $f = \sqrt{0,18} \approx 0,42$. If we estimate the explosive energy as 3,0-4,0 MJ/kg, then equation 43a yields $p_h = 575\text{-}630 \text{ MPa}$.

In both cases equation 43a overestimates Brent's pressures by a factor of 5-6. This amounts to underestimating the effect of decoupling on the bore-hole pressure considerably, at least for high-VOD explosives. The best way to ascertain whose pressure prediction is the most accurate would of course be to measure.

If equation 43a overestimates the pressure in the holes charged with Kimilux, then the true position of the point would move leftward towards the prediction line in Figure 7. Since Brent (1995) doesn't give any data for PETN cord, one can unfortunately not say how the Detonex outlier would move.

Three other ways in which the Kimilux and Detonex points differ are, firstly that for Kimilux one expects a diameter effect on D but that for Detonex one doesn't because the critical diameter is so small, less than 1 mm (Kaye, 1978). Secondly, the VOD of the cord is superseismic with respect to the rock material, $D > c$. This might change the reflection properties at the bore-hole wall when the detonation wave impinges on it. Thirdly, the possible effect of the cord wrapping has already been mentioned.

However, an improvement of the explosives description used here is perhaps best guided by Brent's work (1995). Its two salient features are an effective γ which decreases with decreasing pressure, or density, and a considerably lower pressure level. Using Abel's expansion law for the detonation products (Schmidt, 1935) approximates such behavior by introducing one new material property, the covolume α , which has dimensions m^3/kg . An independent estimate of $\alpha(\rho)$ is given by Johansson and Persson (1970) so the degree of complication of the model itself is small. The prediction equations and the design diagram will however become more complicated.

If this explosives description doesn't improve the curve fit sufficiently, then a further relatively simple, engineering type, calibration is given by Davis (1995). His equation of state for the detonation products contains a smeared out step function that connects the high and low density region behaviors. The simple form presented has 3 new material parameters. Along the principal isentrope, $\gamma \approx 1,3$ in the low density region and a pressure value are the two other parameters required. The ultimate step might well be to use explosives descriptions like Brent (1995) that require advanced computer codes.

The discussion above points to certain deficiencies in the simplified description of the explosive properties which has been used in this report. Each degree of complication distances the model presented here from common engineering use and the number of data behind some points in Figure 7 is quite limited. This suggests that the quality of the underlying data should be improved at the same time as a more complicated explosives description is attempted.

Note though that the results obtained here are far better than the first simple approach presented in Olsson and Bergqvist (1996a), see Figure 3, where the Gurit points do not fall on their prediction line, a line which in itself is dimensionally awkward.

Vånga represents a nearly ideal site to test contour blasting. Thus one could perhaps expect that continued testing would result in an improved crack length prediction, equations 43 and the design diagram. One way to do this would be to stepwise broaden the data base. From a research point of view, complementary tests should then be made, for example:

1. Validate the decoupling influence by testing $\varnothing_e = 17$ mm Gurit in $\varnothing_h = 38$ mm holes and $\varnothing_e = 22$ mm Gurit in $\varnothing_h = 24$ and 38 mm holes.
2. Scrutinize the VOD influence by testing Kimulux 42, $\varnothing_e = 22$ mm in $\varnothing_h = 51$ mm holes and $\varnothing_e = 17$ mm (if available) in $\varnothing_h = 51$ and 64 mm holes. Further, test detonating cord of some other strength than 80g/m.
3. Investigate why the high VOD explosives behave differently.
4. Validate the VOD influence by testing an explosive with $D \approx 3000$ m/s. This opens the field to diluted explosives like Emulet, a styropore diluted AN/emulsion explosive with densities down to 250 kg/m^3 or heavily gassed emulsions deposited in a string so as not to fill the bore-hole.

These data can be used as input in a repeated curve fitting procedure, which is easy to do with existing programs (Deng, 1996). As previously discussed, points 2 to 4 may require an improved explosives description. Thus some of these complementary tests should be instrumented with pressure gauges so that the resulting pressure part of the prediction equations is verified.

Secondly the Vånga test program will during 1997-99 give data on the influence of factors like burden and spacing which can be included in an extended dimensional analysis and curve fitting.

Thirdly, there is a need to investigate the influence of the rock, both in another quarry with relatively fracture free rock with different values for c and K_{lc} , and in rock masses which are more heavily fractured.

The end goal of this work and the Vånga tests is to provide a basis for improved design of cautious blasting. In this light, the present equations and design diagram are only a first, limited step and the actual predictions that they yield must be used with great caution because of the limitations in the analysis.

It would, finally, be expected that the experimental work could be speeded up by using dynamic computations (Song and Kim, 1995 and Minchinton and Lynch, 1996) both to improve the understanding of the complex dynamic phenomena involved, to guide the testing and perhaps even to optimize certain aspects of cautious blasting.

4.2 Conclusions

In this report, mechanics and dimensional analysis are used to derive new prediction equations for the lengths R_c of the radial cracks emanating from the half-casts in the contour after cautious blasting of a contour with a free toe and zero initiation delay between the charges. The influencing parameters are;

- density, VOD and adiabatic expansion exponent of the explosive,
- bore-hole diameter and charge coupling ratio, and
- density, sound velocity and fracture toughness of the rock.

The theory predicts, among other things, that the critical pressure for a dynamic pressurization of a bore-hole depends on hole size, $p_{h,crit} = 1,52 \cdot K_{Ic} / \sqrt{\varnothing_h}$. The experimental data is however best described by the prefactor 3,30, see below.

A number of parameters in the prediction equations are determined by curve fitting to test data obtained from bench blasting in granite in the Vånga quarry. These data do not motivate the use of the full form of the equations however. A compromise between equation complexity and goodness of fit is obtained by

$$p_h = \gamma / (\gamma + 1)^{(\gamma+1)} \cdot p_e D^2 \cdot f^{2,2}, \quad (43a)$$

$$p_{h,crack} = 3,30 \cdot K_{Ic} / \sqrt{\varnothing_h}, \text{ and} \quad (43c)$$

$$2R_c / \varnothing_h = (p_h / p_{h,crack})^{2/[3(D/c)^{0,25} - 1]}. \quad (43b'')$$

The coefficient of determination for these equations is 0,97.

In section 3.2 an approximate design diagram for these equations is given and its use explained. This diagram, see Figure 6, gives a comprehensive view of how the different parameters influence the predicted crack lengths. The end goal of this work is to help provide a basis for improved design of cautious blasting but the prediction tools of this report must be used with great caution because of the limitations of both the analysis and of the underlying data.

A comparison with our Vånga data shows that the prediction equations are very good for the two low VOD explosives used, Gurit and Emulet 20 for which $D \approx 2000$ m/s. It is worse for the two high VOD explosives used, Kimulux 42 and Detonex 80. The crack lengths generated by Kimulux is overestimated by about 70% and those caused by Detonex underestimated by about 25%. A comparison with independent data confirms the prediction quality for two other low VOD explosives and that the crack lengths caused by Detonex are underestimated. This and other referenced work point to some deficiencies in the explosives description which should be considered.

A pure dependence on charge concentration q (kg/m) is not inherent in the prediction equations obtained here. After eliminating p_e and D they reduce to

$$R_c \approx \text{const} \cdot q \cdot e / \varnothing_e^{0.5} \cdot f^{0.7} \text{ for } D = c, \text{ and} \quad (46a)$$

$$R_c \approx \text{const} \cdot (q \cdot e)^{1.5} / \varnothing_e^{1.25} \cdot f^{1.55} \text{ for } D = 0,36 \cdot c, \quad (46b)$$

where e (J/kg) denotes the specific energy of the explosive. For a given explosive the net effect is that $R_c \approx \text{const} \cdot q^{0.75-0.88}$ which is less than the linear relationship postulated by equation 6. On the other hand a low VOD explosive would seem to be relatively more sensitive to decoupling than high VOD one, an observation which is not self-evident.

At the end, tests are recommended and explosives descriptions suggested that could improve the predictive capacity of the equations presented in this report. It should finally be repeated that the present equations and design diagram are only based on an analysis with such limitations that any crack length predictions that can be made for a real, practical blast round must be used with great caution.

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